Radiation Laws and Global Energy Balance

Wien displacement $\lambda_{max} = \frac{a}{T}$ $a = 2{,}898\cdot10^{-3}\,mK$

Stefan-Boltzmann $F = \sigma T^4$ (black body) $F = \varepsilon\sigma T^4$ (grey body)

Planck $B(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(\frac{hc}{\lambda kT})-1}d\lambda$

α: absorbtivity, ρ: reflectivity, τ: transmissivity, ε: emissivity

energy conservation $\alpha+\rho+\tau = 1$

Kirchhoff $\varepsilon_\lambda = \alpha_\lambda$ $S_0 = 1360\,\frac{W}{m^2}$ $\sigma = 5{,}67\cdot10^{-8}\frac{W}{m^2K^4}$

1st order Global Radiation Balance

$\underbrace{\pi R^2 S_0(1-A)}_{in} = \underbrace{4\pi R^2\varepsilon\sigma T^4}_{out} \Rightarrow T = \left(\frac{S_0(1-A)}{4\varepsilon\sigma}\right)^{1/4} \approx -15°C$ (ε=0,95)

πR^2 ⇒ sunlight surface ⇒ $\frac{S_0}{4} = 340\frac{W}{m^2}$ A: albedo (total reflectivity)

2nd order approximation

$A_p \approx 30\%$

surface: $\frac{S_0}{4}(1-A) + \varepsilon_a\sigma T_a^4 = \sigma T_s^4$ ⇒ $T_s^4 = 2T_a^4$

atmosphere: $\varepsilon_a\sigma T_s^4 = 2\varepsilon_a\sigma T_a^4$ ⇒ $T_s = 15°C$

$\varepsilon_a = 0{,}78$ em. up & down

for n layers: $T_s = \left((n+1)\frac{S_0(1-A)}{4\sigma}\right)^{1/4}$ ⇒ Greenhouse effect

$T_s = \left(\frac{S_0(1-A)}{2\sigma(2-\varepsilon_a)}\right)^{1/4}$ $T_a = \left(\frac{S_0(1-A)}{4\sigma(2-\varepsilon_a)}\right)^{1/4}$ $\varepsilon_a \approx 0{,}78$

Earth's fluids

$\partial_z p + g\rho = 0$ / $dp = -g\rho dz$ hydrostatic balance

$pV = nRT$ ideal gas law $R = 8{,}314\,\frac{J}{mol\cdot K}$

$p(z) = p_0e^{-z/H}$ / $H = \frac{R\cdot T_0}{g\cdot M}$ ⇒ $z = H\cdot\ln(\frac{p}{p_0})$ barometric height; scale height

$e_s(T) = e_{s_0}\exp\left(-\frac{L}{R}\left[\frac{1}{T}-\frac{1}{T_0}\right]\right)$ Clausius-Clapeyron ↳ pressure decreased

water: $L = 2{,}501\cdot10^6\,\frac{J}{kg}$ latent heat

$p(z) = p_0 - \rho_w g z$ hydrostatic balance

$\rho(T,S,p) = \rho_0(T)[1+\beta S+\gamma p]$ approximation $S = \frac{mass\ salt}{mass\ solution}$ Salinity

$\rho(T,S=0,p=0) = \rho_{o,max} - 0{,}007\cdot(T-T_{\rho,max})^2$

Fundamentals of Transport

for heat: $D = \frac{\lambda}{c\rho}$ heat conductivity

$\vec j_{ad} = c\cdot\vec v$ advection

$\vec j_{diff} = -D\vec\nabla c$ / $D = \frac13\bar v\lambda$ Fick's law; free path length $\lambda = \bar v\tau$

$\partial_t c = -\vec\nabla\vec j$ general transport eq.

$\partial_t c = -\vec\nabla\vec j_{diff} = \vec\nabla(D\vec\nabla c)$ Fick's 2nd law ⇒ $\partial_t c = D\Delta c$

$\partial_t c = -\vec v\vec\nabla c + D\Delta c$ ⇐ $\vec j$ advection-diffusion eq $[D] = \frac{m^2}{s}$

$\sigma^2 = 2Dt$ diffusion length

advection: motion of eq. heat according to $\vec v$

diffusion: macroscopic expression for microscopic motion

Geophysical Fluid Dynamics

$\tau_{xy} = G\frac{dx}{dy}$ shear stress (solid) $T_{ij} = \begin{pmatrix}\tau_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \tau_{yy} & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \tau_{zz}\end{pmatrix}$

$\tau_{xy} = \mu\frac{dv_x}{dy}$ shear stress (fluid) $T_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)$

$\frac{\partial v_i}{\partial x_j} = \frac12\left[\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right]+\frac12\left[\frac{\partial v_i}{\partial x_j}-\frac{\partial v_j}{\partial x_i}\right]$ strain rate + rot. rate

$\partial_t\rho + \vec\nabla[\rho\vec v] = 0$ continuity eq. (conservation of mass)

$\vec\nabla\vec v = 0$ incompressible fluid ⇒ $\frac{d\rho}{dt} = 0$

$\frac{v^2}{2} + gz + \frac{p}{\rho} = const$ Bernoulli (energy conservation)

$\frac{\mu}{\rho} = \nu$ kinematic viscosity strain rate tensor $\varepsilon_{ij} = \frac12(\partial_j u_i + \partial_i u_j)$

⇒normal stress: compression/tension (fluids & solids)

⇒shear stress: solids deform, fluids flow

⇒ Newtonian fluid: $\frac{du}{dy} \sim \tau$

$\vec v_{Lagrange}(t, \vec x_0(t_0))$: fluid parcel, starting at $\vec x_0(t_0)$

$\vec v_{Euler}(\vec x,t)$: fixed position, $\vec x$ and t independent

$\frac{d_t}{dt} = \partial_t + (\vec v\vec\nabla)$ Lagrange = Euler + advective

Fluid dynamics: EOM

$\vec a = \frac1\rho\cdot\vec f$

$d_t\vec v = \partial_t\vec v + (\vec v\cdot\vec\nabla)\vec v = 0$ Euler eq. $P = \vec F\cdot\vec v$ Work

1) Gravity $\vec a_g = -\vec\nabla\Phi(\vec z,\Phi) = -\vec g$
2) Pressure $\vec a_p = -\frac1\rho\vec\nabla p$ $\vec F_c + \vec F_p = 0$ geostrophy
3) molecular viscous friction $\vec a_r = \nu\Delta\vec v$
4) Coriolis $\vec a_c = -f\hat z\times\vec v$ / $f = 2\Omega\sin\varphi$; $\Omega = \frac{2\pi}{24h}$

↳ F_c right from $\vec v$ (north), left from $\vec v$ (south)

$d_t\vec v = \partial_t\vec v + (\vec v\cdot\vec\nabla)\vec v = -\vec g - \frac1\rho\vec\nabla p - 2(\vec\Omega\times\vec v) + \nu\Delta\vec v$

in components: $\vec v = (u,v,w)$

$\frac{\partial u}{\partial t} + (\vec v\cdot\vec\nabla)u = -\frac1\rho\frac{\partial p}{\partial x} + fv + \nu\Delta u$

$\frac{\partial v}{\partial t} + (\vec v\vec\nabla)v = -\frac1\rho\frac{\partial p}{\partial y} - fu + \nu\Delta v$

$\frac{\partial w}{\partial t} + (\vec v\vec\nabla)w = -g - \frac1\rho\frac{\partial p}{\partial z} + \nu\Delta w$

scale analysis: $\partial_t v \approx \frac{U}{T}$ $(\vec v\cdot\vec\nabla)\vec v \approx U^2/L$ $\nu\Delta\vec v \approx \nu U/L^2$ $\frac1\rho\partial_x p \approx \frac{\Delta P}{\rho L}$ $2\Omega\sin\varphi \approx f\cdot U$ $\partial_x \approx 1/L$ $\partial_t \approx 1/T_c$

$Re = \frac{U\cdot L}{\nu}$ ⇒ laminar or turbulent

$Ro = \frac{U}{fL}$ ⇒ geostrophic if $Ro \ll 1$, inertia dominant if $Ro \gtrsim 1$

$Sc = \frac{\nu}{D}$ ⇒ momentum vs mass diffusion

$Pr = \frac{\nu}{\chi}$ ⇒ momentum vs thermal

vollst. x-Komponente:
$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z} = -g_x - \frac1\rho\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]$

Vertical Motion

Water

$0 = -g - \frac1\rho\frac{\partial p}{\partial z}$ hydrostatic equilibrium

$F_B = g\rho_E V_p$ buoyancy ⇒ vertical acceleration due to density differences

$b = a_z = -g\frac{\rho_p - \rho_E}{\rho_p}$

$b = \frac{g}{\rho}\left(\frac{d\rho}{dz}\right)_E\Delta z$ stratification of water

b>0: $\frac{d\rho_E}{dz} > 0$ unstable ⇒ convection

b=0: $\frac{d\rho_E}{dz} = 0$ neutral

b<0: $\frac{d\rho_E}{dz} < 0$ stable ⇒ oscillation

Boussinesq approximation: small density diff. irrelevant for inertial terms

$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz}$ Brunt-Väisälä frequency (water) $N^2 > 0$ ⇒ stable

Air

$N^2 = \frac{g}{T_E}\left(\frac{dT_E}{dz} - \frac{dT_p}{dz}\right)$ stratification in air

⇒ air more complicated since it is compressible

$\frac{dT}{dz} = -\frac{g}{c_p} = -\Gamma_d \approx -1K/100m$ dry adiabatic lapse rate

$\frac{dT}{dz} = -6{,}5\,K/km$ moist adiabatic lapse rate

$N^2 = \frac{g}{T_E}\left(\frac{dT_E}{dz}+\frac{g}{c_p}\right) = \frac{g}{T}\left(\frac{dT}{dz}+\Gamma_d\right)$ buoyancy f. for dry-adiabatic

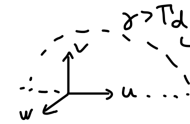
$\theta = T\cdot\left(\frac{p_0}{p}\right)^{\frac{\kappa-1}{\kappa}}$ $\frac{\kappa-1}{\kappa} = \frac{R_d}{c_p} \approx \frac27$ potential temperature: T an air parcel reaches when it is adiabatically compressed to p_0

$N^2 = \frac{g}{\theta}\left(\frac{d\theta}{dz}\right)$ buoyancy frequency

$\frac{d\theta}{dz} > 0$: stable unstable, if $\gamma = -\left(\frac{dT}{dz}\right)_{env}$; $\gamma > \Gamma_d$ ⇒ convection ↳ air parcel is warmer ↳ moves up

$\frac{d\theta}{dz} = 0$: neutral

$\frac{d\theta}{dz} < 0$: unstable



higher pressure, higher water level

Horizontal Motion

$\vec a_c = -2\vec\Omega\times\vec v = \begin{pmatrix}2\Omega v\\ -2\Omega u\end{pmatrix}$

$0 = -\frac1\rho\frac{\partial p}{\partial y} - fu$ geostrophic balance $fu = \frac1\rho\frac{\partial p}{\partial y} \approx \frac1\rho|\nabla p|$

$Ro = \frac{advective}{Coriolis} \sim \frac{U}{fL}$ Rossby number ⇒ neglect non-linear terms for Ro small; ↳ if small ⇒ geostrophic

$u = -\frac{1}{\rho f}\frac{\partial p}{\partial y}$ velocity components $\vec a_{coriolis} = \vec a_{pressure}$

$v = \frac{1}{\rho f}\frac{\partial p}{\partial x}$ $V = \frac{1}{\rho f}\frac{\Delta p}{\Delta x}$ no pressure: $T_0 = \frac{2\pi}{f}$ $r_0 = \frac{u_0}{f}$

⇒flow is along isobars, ⊥ to pressure gradients ⇒circular flow

$\frac{\partial p}{\partial x} = g\rho\frac{\partial h}{\partial x}$ / $\frac{\partial p}{\partial y} = g\rho\frac{\partial h}{\partial y}$ geostrophic flow in ocean

⇒ $u_g = -\frac{g}{f}\frac{\partial h}{\partial y}$; $v_g = \frac{g}{f}\frac{\partial h}{\partial x}$ barotropic: isopycnals & isobars ∥ ; baroclinic: isopycnals & isobars ✗ (density)

h: sea surface elevation

$u(z) = u(0) - \frac{g}{f\bar T}\frac{\partial T}{\partial y}z$ thermal wind

⇒ meridional T gradient ⇒ ρ gradient ⇒ baroclinic p gradient ⇒ flow

$\frac{\partial p}{\partial z} = -\rho g$ ⇒pressure drops more rapidly over colder (denser) air ↳ pressure gradient

⇒northern hemisphere: counterclockwise for low pressure (cyclone); clockwise for high pressure (anticyclone)

Vorticity

SH: clockwise for low pressure (cyclone), counterclockwise for high pressure (anticyclone)

$\vec\xi = \vec\nabla\times\vec v$ vorticity (relative)

$\xi_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ / $\xi = \frac{dC}{dA}$ circulation $C = \oint_{\partial A}\vec v\cdot d\vec l$

$\xi = 2\omega$ rigid rotor

$f = 2\Omega\sin\varphi$ planetary vorticity

$\eta = \xi + f$ absolute vorticity

$\frac{d\eta}{dt} + \eta(\vec\nabla_h\cdot\vec v) = \frac1\rho(\vec\nabla\times\vec f_{ext})$ vorticity eq

$\vec F_p$: high to low; $\vec\nabla p$: low to high

$-\vec\nabla\times\frac1\rho\vec\nabla p = \frac{1}{\rho^2}(\vec\nabla\rho\times\vec\nabla p)$ solenoid term = 0 for barotropic; ≠0 for baroclinic

for ocean:

$\Pi = \frac{\xi+f}{H}$ potential vorticity ↳ fluid column height

$\frac{d}{dt}\left(\frac{f+\xi}{H}\right) = 0$ PV conservation

N: $\xi > 0$ ⇒ cyclonic/anticlock; $\xi < 0$ ⇒ anti-cycl/clock

$Z_R = \frac{\eta}{\frac{\Delta p}{g}}$ Rossby's PV ; $Z_E = \frac{\eta}{\rho}\frac{d\theta}{dz}$ Ertel's PV — atmosphere

⇒Rossby waves: variation of f

⇒ 1 vortex has zero vorticity

⇒ simple shear flow has vorticity

Turbulence

$Re = \frac{inertial\ term}{friction\ term} = \frac{U\cdot L}{\nu}$ Reynolds number turbulent: $Re > Re_c$ L: characteristic length scale (both huge for atm./ocean)

↳ $(\vec v\vec\nabla)\vec v$ produces turbulence, $\nu\cdot\Delta\vec v$ destroys it (viscous friction)

$\vec v(t) = \bar{\vec v}(t) + \vec v'(t)$ with $\bar{\vec v}(t) = \frac1T\int_t^{t+T}\vec v(t)dt$ Reynolds DC

calculus

$f = \bar f + f'$ $\bar{\bar f} = \bar f$ $\overline{fg} = \bar f\bar g + \overline{f'g'}$ (cov)

$\overline{f'} = 0$ $\overline{\bar f g'} = \bar f\overline{g'} = 0$

$Tr(\tau_{ij}^R)$ ⇒ TKE ; $i\neq j$ τ_{ij}^R ⇒ $\rho\overline{v_i'v_j'}$ ⇒ momentum flux

RD + average on $\vec\nabla\cdot\vec v = 0$: $\vec\nabla\bar{\vec v} = 0$, $\vec\nabla\vec v' = 0$

RD + average on NSE:

$\frac{\partial\bar u}{\partial t} + \bar u\frac{\partial\bar u}{\partial x} + \bar v\frac{\partial\bar u}{\partial y} + \bar w\frac{\partial\bar u}{\partial z} = -\frac1\rho\frac{\partial\bar p}{\partial x} + f\bar v + \nu\Delta\bar u - \vec\nabla(\overline{u'\vec v'})$ closure, turb. ↳ parametriz.

$j_{px,y} = (\rho u)v$ ⇒ $\tau_{xy,turb} = \rho\overline{u'v'}$ Reynolds stress

$\tau_{xy,turb} = -\rho\nu_e\frac{\partial\bar u}{\partial y}$ Boussinesq Approx. $\tau_{ij}^R = \rho\overline{v_i'v_j'} = -\rho\nu_e\left(\frac{\partial\bar v_i}{\partial x_j}+\frac{\partial\bar v_j}{\partial x_i}\right)$

$TKE = \frac12\overline{v'^2}$ $E_{kin} = \frac12\rho v^2$ / $\frac{F}{V} = \rho v$ Turbulent kinetic energy

$\varepsilon = -\frac{d(TKE)}{dt}$ energy dissipation rate

$L_K = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$ Kolmogorov scale ⇒ Energy cascade ↳ inertial subrange

$E(k) = c\cdot\varepsilon^{2/3}k^{-5/3}$ Kolmogorov energy spectrum

⇒energy input at large scale ⇒ dissipation at small scale

Turbulent Transport

$j_{turb} = \overline{c'w'}$ turbulent flux only locally; mixing length

$j_{turb,z} = -K_z\frac{\partial\bar c}{\partial z}$; $K_z = \overline{w'l'}$ K-theory/closure; eddy diffusivity

$R_E(\tau) = \frac{\overline{v'(t)\cdot v'(t-\tau)}}{\overline{v'^2(t)}}$ Autocorrelation (memory time)

$R_L(\tau) = \frac{\langle v'(t)\cdot v'(t-\tau)\rangle}{\langle v'^2(t)\rangle}$ $t \ll \tau_L$: $\sigma\propto t$ (ballistic spread.) $t \gg \tau_L$: $\sigma\propto\sqrt t$ (diffusive spread)

$\sigma^2(t) = 2\overline{v'^2(t)}\int_0^t\int_0^{t'}R_L(t'')dt''dt'$ Taylor's theorem

$K = \overline{v'^2}\tau_L = \overline{v'\cdot l'}$ / $l' = v'\tau_L$ $K_z = \frac{\sigma_z^2}{2t}$

⇒turbulent diffusion is anisotropic: $K_h \gg K_z \gg D$ 3D-transport

$\frac{\partial\bar c}{\partial t} = -\bar{\vec v}\cdot\vec\nabla\bar c + \frac{\partial}{\partial x}\left(K_x\frac{\partial\bar c}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial\bar c}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial\bar c}{\partial z}\right) + D\Delta\bar c$

⇒ turbulence has finite memory time τ_L

⇒ tracer release experiments / eddy covariance (direct)

Boundary Layer

⇒boundary: interface where friction matters

⇒no-slip condition: u(z=0)=0

~mm: molecular viscous layer

physics: friction, flow laminar, boundary cond. u(0)=0

shear stress: $\tau = \rho u_*^2$ ⇒ const. with height

horizontal wind:

$\tau = -\rho\nu\frac{du}{dz} = const.$ shear stress

$u(z) = -\frac{\tau}{\rho\nu}z = \frac{u_*^2}{\nu}z$ velocity profile (linear)

1-100m: Prandtl Layer K is property of flow

physics: turbulent friction (eddies), Coriolis negligible

shear stress: $\tau = u_*^2\rho$ ⇒ constant with height

$\tau = \rho\overline{u'w'} = -\rho\overline{w'l'}\frac{\partial\bar u}{\partial z} = -\rho K(z)\frac{\partial\bar u}{\partial z} = const$ (negative)

w': vertical velocity fluct. $K(z) = \overline{w'l'}$ $K(z) = \kappa u_* z$ eddy diffusivity Karman const. κ≈0,4

l': mixing length ~ eddy size ⇒ $l' = \kappa z$

$u_* = \sqrt{\frac{|\tau|}{\rho}}$ friction velocity Def: $\tau = \rho\cdot u_*^2$ $\tilde z = \frac{z\cdot u_*}{\nu}$, $\tilde u = \frac{u}{u_*}$

horizontal wind: $\bar u(z) = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_0}\right)$ Logarithmic wind profile / Law of the wall

100m-1km: Ekman Layer

physics: pressure gradient, Coriolis, turbulent friction

shear stress: decreasing with height $\frac{\partial|\tau|}{\partial z} < 0$

horizontal wind: Spirals to right (NH)

Ekman spiral: greater height ⇒ less friction, higher wind speed ⇒ Coriolis stronger ⇒ wind direction to right (NH) ⇒ parallel to isobars

>1km: free atmosphere

physics: no friction ⇒ geostrophic flow

shear stress: $\tau \approx 0$

horizontal wind: geostrophic $u_g = \frac{-1}{\rho f}\frac{\partial p}{\partial y}$ $v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x}$ u_*: friction velocity z_0: roughness length

other formula: $C_D = \left(\frac{u_*}{u_r}\right)^2 = \left(\frac{\kappa}{\ln(z_r/z_0)}\right)^2$ ↳ wind speed at reference height

⇒near surface (with friction) wind is deflected across isobars toward low pressure region

$\overline{v_i'v_j'} \neq 0$ for $i\neq j$, if correlated $Tr(\tau_{ij}^R) = TKE$

Gas transfer

$c_{s,i} = \lambda_i\cdot p_i = \lambda_i(p-e_s)\cdot x_i$ Henry's law $[j] = \frac{kg}{m^2s}$ p_i: partial pressure in air, x_i: mixing ratio, e_s: saturation water pressure

↳ equilibrium concentration

$j = k(c-c_s)$ gas flux c: bulk water, c_s: surface saturati.

$k = \frac{D}{\delta}$ transfer velocity $[k] = \frac{m}{s}$

$k\propto Sc^{-n}$; $\frac12 \le n \le 1$ δ: thickness of boundary layer D: molecular diffusion coefficient

$Sc = \frac{\nu}{D}$ Schmidt number $\frac{momentum\ diffusivity}{mass\ diffusivity}$

$Pr = \frac{\nu}{\chi}$ Prandtl number $\frac{momentum}{thermal\ diff.}$

$k\propto u_{10}^2$ air: $Sc\approx1$, $Pr\approx1$ j>0 ⇒ from ocean to atm

u_{10}: wind at h=10m water: $Sc\approx1000$, $Pr\approx7$ ⇒ mass diffusion much slower

$p_{H_2O} = e_s = K_H c_w$ water vapour saturation $c_w = \frac{\rho}{M}$

$j_{th} = k\rho c_w\Delta T$

$Q_{net} = Q_{SW} + Q_{LWA} + Q_{LWW} + Q_S + Q_L$ total surface heat flux

(short wave in, long wave from atm in, long wave from water out, sensible (sense) out, latent (e.g. liquid→gas) out)

$Q_S\propto u_{10}(T_W - T_A)$ convection

$Q_L\propto u_{10}(q_s - q_A)$ evaporation ↳ humidity (surface) $\frac{dT}{dt} = \frac1h\frac{Q}{\rho c_w}$ box model

⇒ ocean has immense heat inertia (centuries)

⇒ wind effect: more wind ⇒ more turb ⇒ δ small ⇒ faster gas exchange

⇒ stagnant laminar film model: thin laminar layer of water ⇒ molecular diffusion

⇒ λ bigger for cold water and smaller for salty water

Global Circulation of the Atmosphere

$u(z) = u(0) - \frac{g}{f\bar T}\frac{\partial\bar T}{\partial y}z$ thermal wind

$u = R\Omega\sin\varphi\tan\varphi$ zonal wind speed

westerlies ⇒ to east

⇒ cool air sinks ⇒ high pressure at surface
⇒ wind goes from high pressure to low pressure

$\lambda = \frac{2\pi}{k}$

driver: meridional radiation imbalance
⇒ surplus in tropics, deficit at poles
⇒ we need to transport heat poleward

0°-30°: Hadley Cell up to 30° (⇒ piling effect)

⇒ rise at 0° → move poleward → sink (30°) ↳ return (surface)

aloft: subtropical jet stream (westerly)

surface: trade winds (easterly)

rise: ITCZ (wet, thunderstorms)

sink: dry subtropics

⇒ not one big cell due to cons. angular momentum

30°-60°: Rossby waves ⇒ driven by instabilities (indirect)

⇒ conservation of potential vorticity

$\frac{d\eta}{dt} = 0 \Rightarrow \frac{d\xi}{dt} + \frac{df}{dt} = 0 \Rightarrow \frac{d\xi}{dt} = -\beta v$ (v: northward velocity) $\beta = \frac{df}{dy}$

ridge: poleward bend ⇒ high pressure ⇒ warm $\beta = \frac{2\Omega\cos\varphi}{R}$

trough: equatorward b. ⇒ low pressure ⇒ cold, stormy

stationary waves ⇒ extreme weather

$\omega = uk - \frac{\beta}{k}$ / $c_{ph} = u - \frac{\beta}{k^2}$ / $c_{gr} = u + \frac{\beta}{k^2}$

for $c_{ph} = 0$: $u = \frac{\beta}{k^2} = \frac{\beta\lambda^2}{4\pi^2}$

60°-90°: Polar cell

⇒ sink (90°) ⇒ move equatorward (surface) ⇒ rise 60°

surface: polar easterlies sink: polar high (cold, dry) rise: polar front (storms)

Global circulation of the ocean

⇒ ocean is heated from above ⇒ stable stratification

wind driven

Ekman transport: in NH, net water transport is 90° to the right of wind; in SH, net water transport is 90° to the left of wind

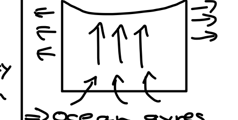
↳ Spirale

wind forcing: anticycl. ⇒ water pushed inward, cycl. ⇒ water pushed outward

⇒ compensation: anticycl: downwelling $w_{Ek} < 0$ ⇒ hill; cycl.: upwelling $w_{Ek} > 0$ ⇒ valley

$w_{Ek} = \frac1\rho\hat z\cdot\left(\vec\nabla\times\frac{\vec\tau_{wind}}{f}\right)$ Ekman velocity

low pressure



⇒ ocean gyres

1) low pressure (NH) ⇒ wind counter-clockwise
2) due to Coriolis force ⇒ net transport of surface water (Ekman layer) is 90° right of the wind
3) divergence of surface water ⇒ outward
4) upwelling ("Ekman suction") from below

$\beta v_g = f\frac{\partial w}{\partial z}$ Sverdrup balance v_g: north-south meridional flow $\frac{\partial w}{\partial z}$: stretching water column

$\beta V = \frac1\rho\hat z\cdot(\vec\nabla\times\vec\tau_{wind})$ Sverdrup relation V: north-south volume transport

Dynamics of other compartments

lakes: water is densest at 4°C ⇒ autumn/spring surface water cools ⇒ sinks ⇒ mixing (brings oxygen)

groundwater

$\phi = \frac{V_{pores}}{V_{total}}$ porosity $\Theta = \frac{V_{water}}{V}$ water content

$h = z + \frac{p}{\rho g}$ hydraulic head h: height water rises to in a well $h = \frac{energy}{weight}$

↳ from Bernoulli

$\vec q = -K\cdot\nabla h$ Darcy's law q: filter velocity K: hydraulic conductivity

in 1D: $q = \frac{Q}{A} = -k\frac{\Delta h}{\Delta l}$ $[q] = \frac{m}{s}$

unconfinend aquifer ↳ h = height ground water confined aquifer ↳ h > height groundwater

⇒ water flows from high h to low hydraulic head

ice:

$e = \lim_{l\to0}\frac{\Delta l}{l}$ strain (deformation) $\varepsilon = \frac{de}{dt}$ strain rate

accumulation zone: gain > loss; ablation zone: loss > gain

$\dot\varepsilon = A|\tau_{eff}|^{n-1}\tau_{ij}$ (strain rate) Glen's law n≈3 ⇒ ice deformation

$\tau_b = \rho gh\sin(\alpha)$ shear stress (glacier) h: ice thickness α: slope angle

$t(z) = -\tau\ln(1-\frac{z}{H})$ simple age model (ice)

Isotope Methods

${}^A_Z X_N$ Z: # protons, N: # neutrons, A: #p+#n

isotone: N=const, isobars: A=const.

$R = \frac{abundance\ heavy}{abundance\ light}$ isotope ratio

isotope fractionation: $v_{rms} = \sqrt{\frac{3kT}{m}}$ ⇒ lighter isotopes diffuse/react faster

⇒ heavier isotopes have higher binding energy ↳ lower zero point energy

$\delta = \frac{R_{sample}}{R_{standard}} - 1$ (·1000‰) δ>0: heavier than standard; δ<0: lighter than standard

$\alpha_{B/A} = \frac{R_B}{R_A}$ fractionation factor (process A→B) α<1: B is lighter; α>1: B is heavier

$\varepsilon_{B/A} = \alpha_{B/A} - 1$ (·1000‰) enrichment ε>0: enrichment; ε<0: depletion

$\delta_B \approx \delta_A + \varepsilon_{B/A}$ $\alpha_{B/A} = \frac{R_B}{R_A} = \frac{1+\delta_B}{1+\delta_A} \approx 1+\delta_B-\delta_A$

$R_A = R_{st}(1+\delta_A)$ $\varepsilon_{B/A} = \alpha_{B/A} - 1 \approx \delta_B - \delta_A$

$\alpha_{B/A} = \frac{1}{\alpha_{A/B}}$ $\varepsilon_{B/A} \approx \delta_B - \delta_A = -\varepsilon_{B/A}$

⇒ light elements fractionate stronger

⇒ $\delta_{solid} > \delta_{liquid} > \delta_{gas}$

⇒ fractionation decreases as temperature increases

⇒ equilibrium (A⇔B) ⇒ reversible $\alpha = K_{eq}$

$K = \frac{[C][D]}{[A][B]}$ law of mass action

$A = \lambda\cdot N$ activity $[A] = Bq = \frac{decays}{s}$

$N(t) = N_0e^{-\lambda t}$ radioactive decay $T_{1/2} = \frac{\ln2}{\lambda}$; $\tau = \frac1\lambda$

$t = \frac1\lambda\ln\left(\frac{N_0}{N(t)}\right)$ age equation 1) N_0 known 2) significant event

Isotopes and Hydrological Cycle

stable isotopes: fractionation ⇒ pathways

radioisotopes: decay ⇒ time scale

primordial (formation solar system), radiogenic (radioactive decays), cosmogenic (cosmic rays), anthropogenic (e.g. bombs)

$\delta^2H = 8\cdot\delta^{18}O + 10‰$ Global meteoric water line

$\delta = \delta_0 + \varepsilon\ln(f)$ Rayleigh fractionation δ: in reservoir f: fraction of reservoir remaining $f = N/N_0$

$R/R_0 = f^{\alpha-1} = f^\varepsilon$ ε: enrichment of process

⇒ temperature effect: colder climate ⇒ precipitation lighter

⇒ continental effect: moving inland ⇒ prec. lighter (progressively)

⇒ altitude effect: moving up a mountain ⇒ prec. lighter

⇒ slope 8: equilibrium fractionation (condensation)

⇒ intercept 10: kinetic fractionation (evaporation)

gyre: large, rotating current system, forms a hill

AMOC: Atlantic Overturning Circulation ⇒ transports heat
⇒ warm water northward (surface)
⇒ cold water southward (deep)

gulf stream: - western boundary current of wind driven gyre
- surface part of AMOC

thermohaline ⇒ density-driven/buoyancy flux ↳ salty

1) polar sinking 2) deep flow 3) upwelling

Incident vs transmitted light

$I(\lambda)=I_0(\lambda)e^{-\tau(\lambda)}$ Beer-Lambert Law τ: optical depth, $\tau=\sigma\cdot N$ (σ cross section)
⇒ when light passes through gas
↳ intensity reduction: absorption & scattering
⇒ Differential Optical Absorption Spectroscopy
↳ scattered light at sunrise/sunset (twilight)
↳ measuring absorption spectrum ⇒ increase in absorption features of stratospheric gases

Climate Sensitivity

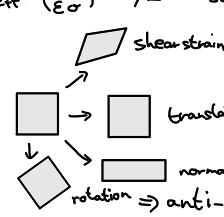
⇒ high λ ⇒ sensitive climate

key feedbacks: $\Delta T=\int^t \frac{F(t')}{C}e^{-\frac{t-t'}{\lambda C}}dt'$
- Planck ⇒ more radiation
+ water vapor (more in warmer air) ⇒ Greenhouse
+ ice albedo ⇒ ice melts ⇒ less albedo
- lapse rate ⇒ vertical
(+) cloud feedback

$\tau=\lambda C$ time dependence C: heat capacity, τ: response time
$F_{CO_2}=5{,}35\ln\left(\frac{C}{C_0}\right)$ ($2\times CO_2$ ⇒ 280 → 560 ppm) measured at top of atmosphere
$\Delta F_{2xCO_2}=3{,}7\ \frac{W}{m^2}$ Radiative forcing
$\lambda=\frac{\Delta T_{eq}}{\Delta F}\left[\frac{K}{W m^{-2}}\right]$ climate sensitivity parameter
$\Delta N=\Delta F+\alpha\Delta T$ climate feedback parameter
at new equilibrium: $\Delta N=0$, $\Delta F=-\alpha T_{eq}$ ⇒ $\lambda=-\frac{1}{\alpha}$
$S=\Delta T_{2xCO_2}=\lambda\Delta F_{2xCO_2}\approx 3°C$ Equilibrium Climate Sensitivity

ocean/atmosphere in total

relative volume increase $K_{thermal}=\frac{\Delta V}{V}\frac{1}{\Delta T}$
box model temperature response $\Delta T=\frac{j\cdot\Delta t}{C}$
weight of atmosphere $p_0=\frac{F}{A}$ ⇒ $F=p_0\cdot A=mg$ ⇒ $m=\frac{p_0A}{g}$
heat capacity of ocean and atmosphere
$C_{atm}=\frac{f+2}{2}R\cdot n_{ath}$ (#moles) $f=5$ (diatomic gas)
$C_{ocean}=C_{H_2O}(A_{earth})\cdot h\cdot\rho_{salt\,water}$ ⇒ without A for C_{column}
$\Delta T=\frac{\Delta F}{C_{H_2O}\rho_w h}t$ $\Delta F=1\frac{W}{m^2}$ ⇒ in 1 umrechnen $\frac{\Delta V}{V}=K_{thermal}\Delta T$
height of earths atmosphere
$p(z)=p_0e^{-z/z_0}$ ⇒ $p(h)=0{,}01\cdot p_0$ ⇒ $h=z_0\cdot\ln\left(\frac{1}{0{,}01}\right)\approx 36{,}8$ km
or detailed: $\rho=\frac{m}{V}=\frac{n\cdot M_{air}}{V}=\frac{p\cdot M_{air}}{R\cdot T}=\frac{p}{R_{air}T}$
$m_h=\int^h\rho(z)A_{earth}dz$
⇒ $m_h=\frac{A_{earth}}{R_{air}T}\int_0^h p_0e^{-z/z_0}dz=-\frac{z_0A_{earth}p_0}{R_{air}T}\left(e^{-h/z_0}-1\right)$
⇒ $m_h=0{,}99\cdot\frac{A_{earth}}{R_{air}T}p_0\int_0^\infty e^{-z/z_0}dz=-\frac{z_0A_{earth}p_0}{R_{air}T}[0-1]$
⇒ $e^{-h/z_0}=0{,}01$
atmospheric heating rate
Energy balance: $I_{in}=I_{ground}+I_{refl.}+I_{atm}$
1) Power = total heat capacity $\cdot\frac{dT}{dt}$ ⇒ $I_{atm}=C_{column}\frac{dT}{dt}$
2) $C_{column}=N_{column}\cdot c_p$ $\boxed{c_p=\frac{7}{2}R}$
$\rho(z)=\rho_{air}\exp(-z/z^*)$ ⇒ $N_{column}=\int_0^\infty\rho(z)dz$
3) convert to daily rate
Longwave radiation
Stefan-Boltzmann: $F=\sigma\varepsilon T^4$ ⇒ less long wave radiation ↳ lower temperature
⇒ T decreases with altitude ⇒ colder radiation must come from higher altitude ⇒ convection, ITCZ, Hadley cell
⇒ radiation from surface blocked by clouds ⇒ emitted ultimately from cold tops of high clouds
$T_{eff}=\left(\frac{F}{\varepsilon\sigma}\right)^{1/4}$, $\Delta T=T_{surface}-T_{eff}$, $z=\frac{\Delta T}{\Gamma}$



shear strain ⇒ off-diagonal elements of symmetric strain rate tensor
translation
normal strain ⇒ diagonal of sym. strain rate tensor
rotation ⇒ anti-symmetric tensor

Venturi meter
1) $p_1+\rho_{pipe}\cdot g\cdot h_L=p_2+\rho_{pipe}\cdot g\cdot h_R+\rho_{mano}\cdot g\cdot\Delta h$
⇒ $\Delta p=p_1-p_2=(\rho_{mano}-\rho_{pipe})g\Delta h$
2) continuity eq: volume flow rate $Q=A\cdot v=const$ $A_1v_1=A_2v_2$ circular pipe: $A=\pi r^2=\frac{d^2}{4}\pi$
3) for horizontal pipe: $p_1+\frac{1}{2}\rho_{pipe}v_1^2=p_2+\frac{1}{2}\rho_{pipe}v_2^2$
↳ insert Δp and $v_1=\frac{A_2}{A_1}v_2$

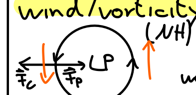
Tool: Box Model

⇒ Don't start with concentration, if volume can change
1) Mass Balance equation
$\frac{dM}{dt}=\sum(\text{Mass flux in})-\sum(\text{Mass flux out})$
advective: $Q\cdot c$ $\frac{dM}{dt}=Q_{in}c_{in}(t)-Q_{out}c(t)-k_rM(t)$
decay: $k\cdot M_i$
inter-box: $k_{ij}M_j$ k_r: reaction rate constant
$k_f=\frac{Q}{V}$: flushing rate
2) $\frac{dM_i}{dt}=V_i\frac{dc_i}{dt}+c_i\frac{dV_i}{dt}$
if volume variable: $\frac{dV}{dt}=\sum Q_{in}-\sum Q_{out}$
ex: $\frac{dc}{dt}=k_fc_{in}-(k_f+k_r)\cdot c$ (mixed reactor) $\Delta^{14}C=-50‰\hat{=}0{,}95$
3) solve ODE   steady state
$c(t)=c_\infty+(c_0-c_\infty)e^{-kt}$ $\boxed{\tau=\frac{1}{k}}$ residence time
with time dependent forcing:
$c(t)=c_0e^{-kt}+\int^t k_fc_{in}(t')e^{-k(t-t')}dt'$ $\boxed{k=k_r+k_f}$

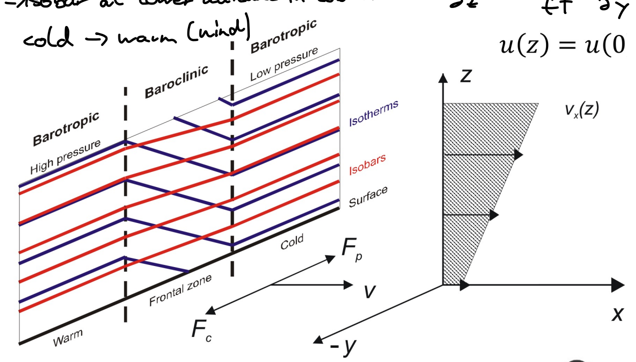
Lake ($V\neq const$) $\frac{dM}{dt}=-q_{out}\cdot c(t)$; $\frac{dV}{dt}=q_{in}-q_{out}=\delta q$
$\frac{dc}{dt}=-\frac{q_{in}}{V(t)}c(t)$; $V(t)=\delta q\cdot t+V_0$ ⇒ $c(t)=c_0\left(\frac{V(t)}{V_0}\right)^{-q_{in}/\delta q}$
2Box-Model
water side: $\frac{dc_w}{dt}=-\frac{j\cdot A}{V_w}=-\frac{A}{V_w}k\left(c_w-\frac{c_a}{\alpha}\right)=-\frac{k}{h}\left(c_w-\frac{c_a}{\alpha}\right)$
air side: $\frac{dc_a}{dt}=\frac{j\cdot A}{V_a}-k_f\cdot c_a=\frac{A}{V_a}k\left(c_w-\frac{c_a}{\alpha}\right)-k_fc_a$ ⇒ $\tau=\frac{h}{k}$
1-Box: $F\cdot c_{Atl}=F\cdot c_{Pac}+\lambda\cdot V_{Pac}\cdot c_{Pac}$ $F=kalt\cdot V_{Atl}$
with residence time τ and decay: $\lambda=\frac{\ln(2)}{T_{1/2}}$
$\frac{dN}{dt}=Q_{in}-N(t)\frac{1}{\tau}-\lambda N(t)$
Be $\frac{dN}{dt}=A\cdot k_f-N(t)k-\lambda N(t)$ ⇒ $\frac{N_{ss}}{A}=\frac{k_f}{k+\lambda}$
diffusion $dM/dt=-j_{PO_4}\cdot A=A\,h\cdot dc/dt$

wind/vorticity/flow

⇒ cold air left for thermal wind



(NH) $\boxed{\eta=\frac{\zeta}{g}+f=const}$ consv. of absolute vorticity
west side: $N\to S$ ⇒ f decreases ⇒ ζ increases
east side: $S\to N$ ⇒ f increases ⇒ ζ decreases
thermal wind
meridional temperature gradients + geostrophic balance
⇒ westerly wind
increase in pressure difference with altitude ⇒ increase in wind speed
⇒ cold air denser ⇒ pressure increases more rapidly with height in cold air
⇒ isobar at lower altitude in cold air $\frac{\partial v_x}{\partial z}=-\frac{g}{fT}\frac{\partial T}{\partial y}$
cold → warm (wind)



⇒ isotherms steeper ⇒ definition of a front
water surface waves $\boxed{k=\frac{2\pi}{\lambda}}$ $v_{Ph}=\frac{\omega}{k}$, $v_{gr}=\frac{d\omega}{dk}$
$c_p=\frac{\omega}{k}=\sqrt{\frac{g}{k}+\frac{\sigma k}{\rho}}$ ⇒ $c_{p,min}$ ⇒ $\frac{dc_p}{dk}=0$ ⇒ $-\frac{g}{k^2}+\frac{\sigma}{\rho}=0$ ⇒ $k_{min}=\sqrt{\frac{g\rho}{\sigma}}$
$f(k)$ ↳ sqrt monotonically increasing
$h(x,t)=h\cos(kx-\omega t)$ ⇒ $a_z=\partial_t^2h$ ⇒ $a_{max}=\omega^2h$ $\boxed{\omega=\sqrt{gk}}$
⇒ $gkh=g$ ⇒ $h=\frac{1}{k}$ ⇒ $\frac{h}{\lambda}=\frac{1}{2\pi}$
⇒ if $a_{max}>g$ ⇒ wave breaks
⇒ if $c\approx\sqrt{gH}$ ⇒ not dependent on k ⇒ $v_{Ph}=v_{gr}$
Film flow on inclined plane
$F=\int^H u_x(y)dy$; $F_{crit}=Re_{crit}\cdot\nu$
Hagen-Poiseuille flow $u_x=u_0\left(1-\left(\frac{r}{R}\right)^2\right)$
⇒ $\tau(r)=-2\mu u_0\frac{r}{R^2}$; $F=\tau(r)\cdot A_{wall}=\tau(r)\cdot 2\pi RL$
⇒ $\tau_{wall}=\frac{2\mu u_0}{R}$ ⇒ max   ⇒ $F_L=\tau_{wall}\cdot 2\pi R=4\pi\mu_0$
⇒ steady: $F_p+F_{Friction}=\Delta p\pi R^2-4\pi u_0\mu\Delta x=0$
estimate thickness of boundary layer $z_0^*=11$
$z_0^*=\frac{z_0\cdot u^*}{\nu_a}$ $u(z)=\frac{u^*}{\kappa}\ln\left(\frac{z}{z_0}\right)$ ⇒ $u^*=\frac{\kappa u_{10}}{\ln\left(\frac{10}{z_0}\right)}$
↳ thickness we want to calc.
Vortex Motion $P_L=\int dA\,\dot{E}_R$ ⇒ mit Jacobi $[\dot{E}]=\frac{J}{s\cdot m^3}$
Velocity field $\frac{d\vec{u}}{dt}=\partial_t\vec{u}+(\vec{u}\cdot\vec{\nabla})\vec{u}$
$dx/dy=u_x/u_y=-\frac{x}{y}$ ⇒ $xy=C$

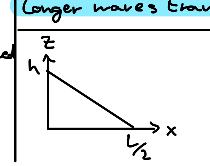
potential temperature

⇒ $\Theta=T\left(\frac{p_0}{p}\right)^{R_s/c_p}$
1. law of thermo $dU=dQ+dW$
adiabatic $dq=c_pdT-\frac{1}{\rho}dp=0$
ideal gas law $pV=nRT$ ⇒ $\frac{1}{\rho}=\frac{R}{M}\frac{T}{p}=R_s\frac{T}{p}$
⇒ $c_p\int\frac{1}{T}dT=R_s\int_p^{p_0}\frac{1}{p}dp$
potential temperature problem
Θ is conserved ⇒ $T_i\left(\frac{p_0}{p_i}\right)^{2/7}=T_f\left(\frac{p_0}{p_f}\right)^{2/7}$
$p_i=p_0e^{-z/z_0}$
advection & turbulent diffusion   characteristic length scale: $e^{-\lambda}=T$
advection-diff.-eq. $\boxed{\frac{\partial T}{\partial t}=-w\frac{\partial T}{\partial z}+K\frac{\partial^2T}{\partial z^2}}$
Ansatz: $T=e^{nz}$ ⇒ $n=0$ or $n=\frac{w}{K}$
rewriting continuity equation
$\rho=\frac{1}{\alpha}$, $\partial_t\left(\frac{1}{\alpha}\right)=-\frac{1}{\alpha^2}\partial_t\alpha$; $\vec{u}\nabla\left(\frac{1}{\alpha}\right)=-\frac{\vec{u}}{\alpha^2}\nabla\alpha$ ⇒ $\frac{d\alpha}{dt}=\alpha\nabla\vec{u}$
Buoyancy $F_B=F_g=g\rho_w\cdot V_d=g\cdot m_d$ $\boxed{\frac{\partial p}{\partial y}=-\rho g\frac{\Delta h}{\Delta y}}$
geopotential height
⇒ higher geopotential height ⇒ higher pressure
evaporation rate $Q_L[W]=L\frac{dm}{dt}$ ⇒ $\frac{Q_L}{L}=\frac{1}{A}\frac{dm}{dt}=\frac{1}{A}\frac{d(\rho Ah)}{dt}=\rho\frac{dh}{dt}$
$E=\frac{h}{t}$ ⇒ $E=\frac{dh}{dt}=\frac{Q_L}{\rho L}$
Gas exchange $k_{CO_2}=k_{heat}\left(\frac{Sc_{heat}}{Sc_{CO_2}}\right)^n$ $j_{momentum}=\rho\cdot u_*$
capture zone of well in uniform flow field
1) Darcy velocity $q=-K\frac{\partial h}{\partial x}$
2) $Q_{min}=w\cdot d\cdot q$
Heat transport into the ground $\bar{T}-\Delta T\,e^{-z/z^*}\geq 0°$
turbulent energy flux in the atmosphere
$je(v)=\frac{1}{2}\rho v^3$; $j_{usable}/j_{total}=\frac{16}{27}$
transport of phosphate $\frac{\partial c}{\partial t}=-\partial_z j_{total}$
$j_{total}=j_{curb}+j_{ad}=-K_z\frac{\partial c}{\partial z}+cw$
isotope fractionation evaporation
evaporation line has slope less than 8 ⇒ slope $=\frac{\delta^2H_f-\delta^2H_i}{\delta^{18}O_f-\delta^{18}O_i}$
⇒ involves equilibrium and kinetic fractionation
oxygen isotope ratio $\delta^{18}O$
1) when water evaporates from ocean surface, lighter molecules evaporate more readily ⇒ vapor depleted in ^{18}O ($\delta<0$)
2) transport towards poles ⇒ as it cools ⇒ precipitation
3) ice forms ⇒ isotopically light
⇒ increase in amount of continental ice leads to increase in oceanic ^{18}O
Mass balance: $M_{tot}\cdot\delta_{tot}=M_{ocean}\cdot\delta_{ocean}+M_{ice}\cdot\delta_{ice}$
and $\delta_{o,new}=\frac{M_{ice}\cdot\delta_{ice}}{M_o+M_{ice}}$ $V_i=\pi R^2\cdot 3800$m
derive $\delta^{18}O_{mix}=x\cdot\delta^{18}O_A+(1-x)\cdot\delta^{18}O_B$ $x=\frac{N_A}{N_A+N_B}$
Ansatz: $R=\frac{n_h}{n_l}$ ⇒ $n_h=n_l\cdot R$ $n_l\propto N$ $1-x=\frac{N_B}{N_A+N_B}$
⇒ $R_{mix}\cdot S_{mix}=R_A\cdot N_A+R_B\cdot N_B$; $R_{mix}=R_{st}(1+\delta_{mix})$
Temperature effect
$\frac{d\delta_p}{dT}=\frac{d\delta_v}{dT}=\left(\frac{d\delta_v}{df}\right)\left(\frac{df}{dT}\right)=\left(\frac{\varepsilon_{lv}}{f}\right)\left(\frac{1}{e_s(T_0)}\frac{de_s}{dT}\right)=\frac{\varepsilon_{lv}L_v}{R_vT^2}$
Atmospheric $^{14}C/^{12}C$ ratio $^{14}N=\frac{x\cdot N_A\cdot V\cdot\rho}{M_{mol}}\cdot 10^{-12}$
Suess effect: dilution by fossil fuels (only ^{12}C)
calculate variation (e.g. of T_s)
$T_s=\left(\frac{S_0(1-A)}{2\sigma(2-\varepsilon_a)}\right)^{1/4}$ $\frac{dT_s}{dS_0}=\frac{1}{4}\left(\frac{(1-A)}{2\sigma S_0^3(2-\varepsilon_a)}\right)^{1/4}$ $\left(\frac{\Delta R}{R}\approx\varepsilon\right)$
⇒ $\frac{\Delta T_s}{T_s}=\frac{1}{4}\frac{\Delta S_0}{S_0}$
cooling rate of stratospheric air
1) $\frac{dT}{dt}=\Gamma\cdot w$ (50 m/d)
2) $P=\rho\cdot H\cdot c_p\cdot\frac{dT}{dt}=\rho\cdot g\cdot H\cdot w$
Hydrostatic equilibrium
water: $\frac{dp}{dz}=\rho g$ ⇒ $\int_{p_0}^p dp'=\int_0^z\rho g\,dz$ ⇒ $p(z)=p_0+\rho gz$
air: $\frac{dp}{dz}=-\rho g=-\frac{pM}{RT}g\,dz$ ⇒ $p(z)=p_0e^{-z/H}$; $H=\frac{RT}{gM}$
Cyclones and Anticyclones $\frac{\partial p}{\partial r}$ is max for $\frac{\partial^2p}{\partial r^2}=0$ ⇒ $r=\frac{R}{\sqrt{2}}$

Aquifer water balance problem $Q_{in}=R\cdot A_{recharge}=Q_{out}=v\cdot A_{cross}$
Heat flux from lake Constance $Q_{therm}=c_p\cdot\Delta T\cdot\rho_w\cdot A\cdot h$
$E=\frac{0{,}4\,W_{net}}{L\cdot\rho}$ $W_{net}=\frac{Q_{therm}}{t\cdot A}$
Stability
ascending air from z_0 to z_1 adiabatically ⇒ no loss of energy ⇒ Θ is conserved
⇒ since rising air is always warmer than the ambient air ⇒ convection is possible ⇒ at z_1 ⇒ same potential temperature
⇒ rising stops

Verständnisfragen

HDO lower water vapor pressure
⇒ HDO sits deeper in energy well ⇒ harder to break bonds
⇒ harder to evaporate
vertical pressure gradient steeper in cold air
$\frac{dp}{dz}=-\rho g$, $\rho=\frac{p}{R_sT}$
isotopes: rain is more enriched than vapour
⇒ colder ⇒ more fractionation but lighter rain overall (more north and vapour is even more negative)
Ekman layer (ocean): changes depth, deeper at equator
$d=\pi\cdot\sqrt{\frac{2K(z)}{|f|}}$ ⇒ ca. 44m at 45° N
buoyancy oscillation $\frac{\partial\Theta}{\partial z}>0$ stable ⇒ potential temperature increases with altitude
1) parcel lifted up ⇒ cools down ⇒ becomes denser ⇒ sinks back
2) parcel pushed down ⇒ warms up ⇒ less dense ⇒ rises
⇒ oscillation, if $\frac{\partial\Theta}{\partial z}<0$ it would continue acceleration
global atm-ocean heatflux   highly variable and uneven
→ tropics receive more solar energy than poles
⇒ large scale temperature gradients
⇒ e.g. gulf stream (heat loss from ocean), heat gain exist as well
atmospheric low implies vertical velocity in ocean
⇒ cyclone ⇒ Ekman transport out of center ⇒ upwelling / Ekman Suction
deriving geostropic flow
1) no acceleration $\frac{du}{dt}=\frac{dv}{dt}=0$
2) no friction
p=const. λ increases linearly with T ⇒ False
$D=\frac{1}{3}\lambda\bar{v}$; $E_{kin}=\frac{1}{2}m\bar{v}^2=\frac{3}{2}k_BT$ ⇒ $D\propto\sqrt{T}$
shear stress ⇔ momentum flux density
heavier ⇒ lower water vapour pressure
why is ocean more stable?   atmosphere ⇒ convection
Ocean: heating ⇒ thermocline ⇒ stable
greenhouse gases
50% of emitted CO_2 since preindustrial times stayed in atmosphere
⇒ water vapour most important
grid box $v=100\frac{m}{s}$ → t smaller than 1 step
molecular transport properties $D_{mass}\approx D_{heat}\approx\nu$ (kin. viscosity momentum)
$P_r\approx Sc\approx 1$
Coriolis is weakest at the equator $\sin(0°)=0$
stress $\tau\propto\varepsilon$ (strain rate)   only true for Newtonian fluids
negative lapse rates and Θ Lapse rate is rate of decrease with height
↳ increase with height $\frac{d\Theta}{dz}>0$
atmosphere vs ocean   flow in atmosphere: compressible $p=R_s\rho T$
flow in ocean: incompressible
↳ liquids are hard to compress
fluids vs solids (shear stress)
Solids: deformation (elastically ⇒ equilib. between stress and strain) or to final deformation state (might break)
fluids: flow, non-elastic deformation ⇒ equilib. between stress and strain rate is reached
derive div $\vec{v}=0$
continuity eq. $\frac{\partial\rho}{\partial t}=-\vec{\nabla}(\rho\vec{v})=-\vec{\nabla}\vec{j}_{mass}$
$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+\vec{v}\cdot\vec{\nabla}\rho=0$
thermal diffusion   heavier molecules go to colder end of tube
Groundwater flow direction is only in isotropic media opposite to gradient of hydraulic head
inertial oscillations   Coriolis = inertia
Groundwater dating $V=\frac{N_{total}}{c_s}$ $N_{total}=\frac{^{81}N_{kr}}{R_a}$, $^{81}N_{kr}=\frac{1000}{\eta}$, $c_s=\lambda(p-e_s)x$
⇒ $V=\frac{N_{total}}{N_A\cdot c_s}$
if we are interested in decays: $A=\lambda\cdot N$, $A=\frac{\Delta N}{\Delta t}$ ← das wollen wir
$N=\frac{T_{1/2}\cdot A}{\ln(2)}$; $N_{total}=\frac{N}{R_a}$; $V=\frac{N}{N_A\cdot c_s}$
earths atmosphere not isothermal
troposphere (0-12km): T decrease with height ⇒ heated from ground
↳ air parcels rise and cool adiabatically
stratosphere (12-50km): T increase with height ⇒ heated from above by absorption of sun's UV by ozone layer
mesosphere and thermosphere: mixed
moist vs dry adiabatic lapse rate
⇒ condensation & latent heat ⇒ as parcel rises it cools and can't hold as much water vapor ⇒ condensation ⇒ releases heat ⇒ warms parcel
Longer waves travel faster than shorter waves



Windmill: $\frac{d}{dx}\ln\left(\frac{x}{y}\right)=\frac{1}{x}$
$E=\eta\cdot\pi r^2\cdot\frac{1}{2}\rho u^2\cdot u$

Einheiten

$[F_{Stefan\text{-}B}]=\frac{W}{m^2}=\frac{J}{Sm^2}$ $J=\frac{m^2kg}{s^2}$ $1bar\hat{=}100.000$ Pa
$[p]=Pa=\frac{N}{m^2}$ $N=\frac{kg\,m}{s^2}$ $1hPa=100Pa$
$[n]=$#moles $[P]=W=\frac{J}{S}$ $[c_p]=\frac{J}{kg\cdot K}$ $1ppm\hat{=}10^{-6}$
$[C_{heat}]=\frac{J}{K}$ $[D]=\frac{m^2}{S}$ $[\dot{E}]=\frac{J}{Sm^3}$
$[\nu]=\frac{m^2}{S}$ kinematic viscosity
$[\mu]=Pa\cdot s$ dynamic viscosity
$[\tau]=\frac{N}{m^2}=Pa$ $1Sv=10^6\frac{m^2}{S}$
$[\dot{\varepsilon}]=\frac{1}{S}$
$[N_A]=\frac{1}{mol}$ continuity warmth $\frac{\partial W}{\partial t}+\nabla\vec{j}_{th}=Q$
$[j_{mass\,flux}]=\frac{kg}{m^2S}$ $12u\hat{=}12\frac{g}{mol}$
$[j_{heat\,flux}]=\frac{W}{m^2}$ $1L\hat{=}0{,}001\ m^3$
$[\text{concentration}]=\frac{mol}{kg}/\frac{mol}{L}/\frac{kg}{m^3}$ $1\frac{g}{cm^3}\hat{=}1000\frac{kg}{m^3}$
$[\rho]=\frac{kg}{m^3}/\frac{mol}{m^3}$
$p=\frac{F}{A}$ pressure $A_{earth}=4\pi R_{earth}^2$

$$\boxed{\begin{aligned}n&=\frac{N}{N_A}\\ m_{Total}&=n\cdot M_{mol}\\ c&=\frac{n}{V}\\ \rho&=c\cdot M_{mol}\end{aligned}}$$

Thermo

$T_1=T_0\left(\frac{P_0}{P_1}\right)^{\frac{1-K}{\gamma}}$ P-T relation (adiabatic) $f=3$ / $f=5$
$\gamma=\frac{c_p}{c_v}$ $c_p=c_v+R_d$; $\gamma=\frac{f+2}{f}$, $K=-\frac{1-\gamma}{\gamma}$
$\boxed{dU=dQ+dW}$ 1st law of thermodynamics
1) $dU=c_vdT$, $dW=-pdv$ ⇒ $c_vdT=dQ-pdv$
2) adiabatic: $dQ=0$ $d(pv)=pdv+vdp=R_sdT$
↳ no heat exchange
$\boxed{pV=nRT}$ ideal gas $\frac{p}{\rho}=pv=\frac{R}{M}\cdot T=R_sT$ $v=\frac{1}{\rho}$
⇒ $pdv=R_sdT-vdp$
⇒ $c_vdT+R_sdT-vdp=0$ ⇒ $\boxed{c_pdT-\frac{1}{\rho}dp=0}$
⇒ $dp=-g\rho dz$ for adiabatic lapse rate

$dq=c_pdT-vdp=0$

$\vec{\nabla}\times\vec{F}=\begin{pmatrix}\frac{\partial F_3}{\partial x_2}-\frac{\partial F_2}{\partial x_3}\\ \frac{\partial F_1}{\partial x_3}-\frac{\partial F_3}{\partial x_1}\\ \frac{\partial F_2}{\partial x_1}-\frac{\partial F_1}{\partial x_2}\end{pmatrix}$ $R_{s,Luft}=\frac{R}{M}=\frac{8{,}314\ J/mol\cdot K}{0{,}029\ kg/mol}=287\frac{J}{kg\cdot K}$

critical Re
circular pipe ~2300
flow over plate ~500.000
rivers/canals ~ 1000
Material derivative
$\frac{da}{dt}=\partial_ta+v_i\partial_ia=\frac{\partial a}{\partial t}+\frac{\partial a}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial a}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial a}{\partial z}\frac{\partial z}{\partial t}$
$\frac{dg_i}{dt}=\partial_tg_i+v_i\partial_ig_j=\frac{\partial\vec{g}}{\partial t}+\left(\frac{\partial g_x}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial g_x}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial g_x}{\partial z}\frac{\partial z}{\partial t}\right)\hat{e}_x+\dots$

Values

$R=8{,}314\frac{J}{mol\cdot K}$ $\rho_{ice}=917\frac{kg}{m^3}$, $\tau_{0,ice}=100.000$ Pa
$\rho_{salt\,water}=1030\frac{kg}{m^3}$
$e_s(20°C)=23{,}4$ hPa
$p_0=1013$ hPa $e_{s,0}=611$ Pa
$c_{p,air}=1004\frac{J}{kg\cdot K}$ $M_{H_2O}=0{,}018015\frac{kg}{mol}$
$V_{ocean}=1{,}4\cdot 10^{18}$ m $T_0=273{,}15$ K

Dependencies

dynamic/kinematic viscosity
gases: $\mu=\mu(T)$ incr. with T
Liquids: $\mu=\mu(T,p)$ incr. with T, decr. with p
density
gases: $\rho=\frac{p}{R_sT}$ → $\rho(p,T)$ ← salinity
Liquids (water): $\rho=\rho(T,S,p)$
gas solubility
$\lambda=\lambda(T,S)$ decr. with T/S
saturation water vapor pressure
$e_s=e_s(T)$ incr. with T
molecular diffusion coefficient
$D=D(T,\nu,\text{fluid},\text{tracer})$

$e^{a\cdot\ln(b)}=e^{\ln(b^a)}=b^a$
$\sinh(x)=\frac{1}{2}(e^x-e^{-x})$
$\cosh(x)=\frac{1}{2}(e^x+e^{-x})$
$\lambda=2\pi R\cos\phi$

Integrale
$\int dV_{Kugel}=\int_0^\infty dr\int_0^{2\pi}d\varphi\int_0^\pi d\theta\ r^2\sin\theta$
$\int dV_{Zylinder}=\int_0^\infty dz\int_0^{2\pi}d\varphi\int_0^\infty dr\ r$

$Re=Re(U,L,\nu)$ K/ν_e is property of flow, not fluid
$f=f(\varphi)$ $K=K(U,L,z,\text{stability})$
$\beta=\beta(\varphi)$ gas transfer velocity: $k=k(D,S)=k(u_{10},D,T)$
$V_g=v_g(f,\text{slope})$ $\frac{\partial v_g}{\partial z}(f,T,\nabla T)$

Transport $\frac{\partial c}{\partial t}=D\partial_z^2c-\lambda c=0$; Ansatz: $c(z)=e^{kz}$
B.C. 1) $\left.\frac{dc}{dz}\right|_{z=H}=0$; 2) $\left.\frac{dc}{dz}\right|_{z=0}=-\frac{j_0}{D}$
Trick: $\frac{j_0\ e^{kH}}{Dk(1-e^{-2kH})\cdot e^{kH}}$