$\dot p_i = q\left(E_i + \frac{1}{c}\varepsilon_{ijk} v^j B^k\right)$ Lorentzgleichung

## Maxwell-Gleichungen

M1 $\partial_i D^i = 4\pi\rho$ Gauß-Gesetz

M2 $\partial_i B^i = 0$ Fehlen magn. Lad.

M3 $\varepsilon^{ijk}\partial_j E_k = -\partial_{ct} B^i$ Faraday-Gesetz

M4 $\varepsilon^{ijk}\partial_j H_k = \partial_{ct} D^i + \frac{4\pi}{c} j^i$ Ampère-Gesetz

$\int_V dV\, \partial_i j^i = \int_{\partial V} dS_i\, j^i$ Satz von Gauß

$\int_S dS_i\, \varepsilon^{ijk}\partial_j E_k = \oint_{\partial S} dr^i E_i$ Satz von Stokes

M1 $\int_V dV \partial_i D^i = \int_{\partial V} dS_i D^i = \psi = 4\pi \int_V dV \rho = 4\pi q$

$\int_S dS_i D^i = 4\pi r^2 D = 4\pi q \Rightarrow D = \frac{q}{r^2}$

M2 $\int_V dV \partial_i B^i = \int_{\partial V} dS_i B^i = \phi = 0$

M3 $\int_S dS_i \varepsilon^{ijk}\partial_j E_k = U = \oint_{\partial S} dr^i E_i = -\frac{d}{d(ct)}\int_S dS_i B^i$

M4 $\int_S dS_i \varepsilon^{ijk}\partial_j H_k = \oint_{\partial S} dr^i H_i = + \frac{d}{d(ct)}\int_S dS_i D^i + \frac{4\pi}{c}\int_S dS_i j^i = + \frac{d\psi}{d(ct)} + \frac{4\pi}{c} I$

## Felder in Materie

isotrop: $\mu_{ij} = \frac{1}{\mu}\gamma_{ij}$, $\varepsilon_{ij} = \frac{1}{\varepsilon}\gamma_{ij}$, $\mu^{ij} = \mu\gamma^{ij}$, $\varepsilon^{ij} = \varepsilon\gamma^{ij}$

$D^i = \varepsilon^{ij}E_j \Leftrightarrow E_i = \varepsilon_{ij} D^j$

$B^i = \mu^{ij} H_j \Leftrightarrow H_i = \mu_{ij} B^j$

## Ladungserhaltung

$\partial_{ct} = \frac{1}{c}\partial_t$

$\partial_t \rho + \partial_i j^i = 0$ Kontinuitätsgleichung

Divergenz auf M4 anwenden

$\frac{d}{dt}\int_V dV \rho = \frac{d}{dt} q = -\int_V dV \partial_i j^i = -\int_{\partial V} dS_i j^i = -I$

## Elektrostatisches Potential

$\Delta\phi = -4\pi\rho$ Poisson

$E = \frac{q}{r^2} \Rightarrow \vec E(\vec r) = -\nabla\phi(\vec r)$

$\phi(\vec r) = \int dV' \frac{\rho(\vec r')}{|\vec r - \vec r'|}$  $\varepsilon^{ij}\partial_i\partial_j\phi = -4\pi\rho$

$E_i = -\partial_i\phi$

$\Delta \frac{1}{|\vec r - \vec r'|} = -4\pi\delta(\vec r - \vec r')$ in M1

$w_{el} = \frac{\varepsilon_{ij} D^i D^j}{8\pi}$, $w_{mag} = \frac{\mu_{ij} B^i B^j}{8\pi}$, $\tilde G = \frac{4\pi}{k^2}$

Stetigkeit: $D_2^\perp = D_1^\perp + 4\pi\sigma$, $E_2^\parallel = E_1^\parallel$

## Maxwell-Gleichungen

M1 $\nabla\vec E = 4\pi\rho$  $i\vec k\vec E = 4\pi\rho$

M2 $\nabla\vec B = 0$  $\vec k \vec B = 0$

M3 $\nabla\times\vec E = -\partial_t \vec B$  $\vec k\times\vec E = \frac{\omega}{c}\vec B$

M4 $\nabla\times\vec B = \partial_t\vec E + \frac{4\pi}{c}\vec j$  $i\vec k\times\vec B = -i\frac{\omega}{c}\vec E + \frac{4\pi}{c}\vec j$

## Fouriertrafo

$\tilde f(\omega,\vec k) = \int d^3x\,dt\, f(t,\vec x) e^{i(\vec k\vec x - \omega t)}$

$f(t,\vec x) = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} d^3k\, d\omega\, \tilde f(\vec k,\omega) e^{-i(\vec k\vec x - \omega t)}$

$\nabla \to i\vec k$, $\Delta \to -k^2$, $\partial_t \to -i\frac{\omega}{c}$

$\langle e^{i\vec k\vec x}, e^{i\vec k'\vec x}\rangle = (2\pi)^n \delta^{(n)}(\vec k - \vec k')$

$\langle f, g\rangle = \int d^n x\, f(x) g^*(x)$

## Kovarianter Indexkram

stumme Indizes umbenennen

$\frac{\partial(a_\alpha A_\alpha)}{\partial(a_\mu A_\nu)} = \delta^\alpha_\mu\delta^\alpha_\nu$, $\frac{\partial F_{\gamma\sigma}}{\partial F_{\mu\nu}} = \delta^\mu_\gamma\delta^\nu_\sigma$, $\frac{\partial x^\mu}{\partial x^\mu} = 4$

$\eta_{\mu\nu}\Lambda^\mu_\alpha\Lambda^\nu_\beta = \eta_{\alpha\beta}$, $\Lambda^\mu_\alpha\Lambda^\alpha_\beta = \delta^\mu_\beta$

$\partial_\mu x^\nu = \frac{\partial x^\nu}{\partial x^\mu} = \delta^\nu_\mu$

$\partial^\mu x_\nu = \frac{\partial x_\nu}{\partial x_\mu} = \delta^\mu_\nu$, $\partial^\mu x^\nu = \frac{\partial x^\nu}{\partial x_\mu} = \eta^{\mu\nu}$

## Indexkram

$\delta^i_i = 3$  $\Delta = \gamma^{ij}\partial_i\partial_j$

Vektor $\vec v = v^i e_i$; $e_i e^j = \delta_i^j \Rightarrow \vec p \vec v = p_i v^i$

Linearform $\vec p = p_i e^i$  $\gamma^{ij}\gamma_{jk} = \delta^i_k$

Metrik $v_i = \gamma_{ij} v^j$  $\vec p \cdot \vec v = \gamma^{ij} p_i v_j$

$(\vec a \times \vec b)^i = \gamma^{il}\varepsilon_{ljk} a^j b^k$, $(\mathrm{rot}\,\vec a)_i = \varepsilon_{ijk}\gamma^{jl}\partial_l a^k$, $\frac{\partial x^i}{\partial x^j} = \delta^i_j$

$\mathrm{div}\,\vec a = \partial_i a^i$

$\partial_i = \frac{\partial}{\partial x_i}$, $\frac{\partial r}{\partial x_i} = \frac{x_i}{|\vec x|}$

Grassmann: $\varepsilon^{kij}\varepsilon_{klm} = \delta^i_l\delta^j_m - \delta^i_m\delta^j_l$

$\vec a\times(\vec b\times\vec c) = (\vec a\cdot\vec c)\vec b - (\vec a\cdot\vec b)\vec c$

$\nabla\times(\vec c\times\vec d) = (\vec d\nabla)\vec c - (\vec c\nabla)\vec d + \vec c(\nabla\vec d) - \vec d(\nabla\vec c)$

$\mu_{kl}\varepsilon^{ijk}\varepsilon^{lmn} = \mu^{im}\mu^{jn} - \mu^{in}\mu^{jm}$

## Potentialtheorie - Statik

$\phi(\vec k) = \frac{4\pi}{k^2}\rho(\vec k) \Rightarrow G(\vec k) = \frac{4\pi}{k^2}$

$0 = \int d^3x\,(\phi_1 - \phi_2)\partial_i\partial^i(\phi_1-\phi_2) = \int_{\partial V} dS_i (\phi_1-\phi_2)\partial^i(\phi_1-\phi_2) - \int_V d^3x\, \partial_i(\phi_1-\phi_2)\partial^i(\phi_1-\phi_2)$, $\rho_1 - \rho_2 = 0$

$\int dV' (\gamma^{ij}\partial_i\phi\partial_j\psi + \phi\Delta\psi) = \int_{\partial V} dS_i\, \gamma^{ij}(\phi\partial_j\psi)$ 1. Green

$\int dV'(\phi\gamma^{ij}\partial_i\partial_j\psi - \psi\gamma^{ij}\partial_i\partial_j\phi) = \int_{\partial V} dS_i\,\gamma^{ij}(\phi\partial_j\psi - \psi\partial_j\phi)$ 2. Green

Neumann: $n_i\partial^i\phi(\vec r)|_{\partial V}$ fest $\Rightarrow (\phi_1-\phi_2)|_{\partial V} = 0$, Dirichlet: $\phi(\vec r)|_{\partial V} \Rightarrow dS_i\partial^i(\phi_1-\phi_2)|_{\partial V} = 0$

$W_{el} = \frac12\int dV \rho(\vec r)\phi(\vec r)$ äußeres Potential, Probeladung  $\partial_i\partial_j\phi - \frac{\Delta\phi}{3}\gamma_{ij}$ traceless

$\Rightarrow W_{el} \cong \frac12 q\phi(\vec r) + \frac12 q^i\partial_i\phi + \frac{1}{6} q^{ij}\partial_i\partial_j\phi$

$\nabla\vec B = 0 \Rightarrow \exists\vec A: \vec B = \mathrm{rot}\,\vec A$; $\mathrm{rot}\,\vec E = 0 \Rightarrow \exists\phi: \vec E = -\nabla\phi$

## Eichung & magnetisches Vektorpotential

Magnetostatik $\Rightarrow$ M3  $\varepsilon^{ijk}\partial_j H_k = \frac{4\pi}{c}j^i \Rightarrow B^i = \varepsilon^{ijk}\partial_j A_k \Rightarrow$ M2 aut. erfüllt

Coulomb-Eichung $\gamma^{ij}\partial_j A_i = 0$, $A_i \to A_i + \partial_i\chi$  $\Rightarrow \Delta A_i = -\frac{4\pi\mu}{c}\gamma_{ij}j^j$

$\Rightarrow \Delta\chi = -\gamma^{ij}\partial_i A_j \Rightarrow$ divergenzfrei

$\hookrightarrow$ zur Poisson ähnliche Gleichung in Magnetost.

## Dynamik - Elektromagnetische Felder

hom. MG (E,B) $\Rightarrow \exists$ Potentiale; inhom. MG (D,H) $\Rightarrow$ Felder $\Leftrightarrow$ Ladungen

$A_i$ in M3: $E_i = -\partial_i\phi - \partial_{ct}A_i$; $\phi \to \phi - \partial_{ct}\chi$; $A_i \to A_i + \partial_i\chi$

$\Delta\phi = -4\pi\rho$, $\phi(\vec r) = \int dV' \frac{\rho(\vec r')}{|\vec r - \vec r'|}$

$\Delta A_i = \frac{4\pi}{c}\gamma_{ij}j^j$, $A_i(\vec r) = \int dV' \frac{j^j(\vec r')}{|\vec r - \vec r'|}\gamma_{ij}$

$A_i, \phi$ in M1, M3 $\Rightarrow$ Lorenz-Eichung $\varepsilon\partial_{ct}\phi + \frac{1}{\mu}\gamma^{ij}\partial_i A_j = 0$

d'Alembert Operator $\Box = \varepsilon\mu\partial_{ct}^2 - \Delta$  $\Box\phi = \frac{4\pi}{\varepsilon}\rho$, $\Box A_i = \frac{4\pi}{c}\mu\gamma_{ij}j^j$

$c' = \frac{c}{\sqrt{\varepsilon\mu}} \Rightarrow n = \sqrt{\varepsilon\mu}$ Brechungsindex, $\Box\chi = \varepsilon\mu\partial_{ct}^2\chi - \Delta\chi = \varepsilon\mu\partial_{ct}\phi + \partial_i A^i$

Wave eq. Coulomb-Eichung: $\Delta\phi = -\frac{4\pi}{\varepsilon}\rho$, $\Delta A_i - \partial_{ct}^2 A_i = -\frac{4\pi}{c}\mu\gamma_{ij}j^j + \varepsilon\partial_i\partial_{ct}\phi$

## Liénard-Wiechert-Potentiale

Lorenz-Eichung: Wellen $\Rightarrow \Box\phi = 4\pi\rho$, $\Box A_i = \frac{4\pi}{c}\gamma_{ij}j^j$; Vakuum $\Rightarrow \Box\phi = 0$, $\Box A_i = 0$

Ansatz: ebene Wellen $\Rightarrow \phi, A_i \propto \exp(\pm i(\omega t - k_i r^i)) \Rightarrow$ Dispers.rel. $\omega = \pm ck$

$v_{Ph} = \frac{\omega}{k} = c = \frac{d\omega}{dk} = v_{gr}$ $\Rightarrow$ Wellen mit Geschw. c in Lorenz-Eichung, dispersionsfrei

im Vakuum: $\varepsilon^{ijk}\partial_j E_k = -\partial_{ct}B^i$, $\varepsilon^{ijk}\partial_j H_k = +\partial_{ct}D^i$; $\gamma^{ij}k_i E_j = 0$, $\gamma^{ij}k_i H_j = 0$

Dualitätstrafo $E_i \to H_i$, $H_i \to -E_i$

für homogenes Feld: $B = \text{const} \Rightarrow \partial_i B^k = 0$

Ohmsches Gesetz $j^i = \sigma E_i$

Telegraphengleichung $\Box H_i = -\frac{4\pi\sigma\mu}{c}\partial_{ct} H_i$ mit $c' = \frac{c}{\sqrt{\varepsilon\mu}}$  $\omega = ck$

## Kugelkoordinaten

$x = r\cos\varphi\sin\theta$, $y = r\sin\varphi\sin\theta$, $z = r\cos\theta$, $|J| = r^2\sin\theta$

$V = \int_0^{2\pi}d\varphi\int_0^\pi d\theta\int_0^R dr\, r^2\sin\theta$

Zylinder: $x = r\cos\varphi$, $y = r\sin\varphi$, $z = z$, $|J| = r$; $\int d\varphi\int dr\int dz\, r$

$\vec e_r = \begin{pmatrix}\sin\theta\cos\varphi\\ \sin\theta\sin\varphi\\ \cos\theta\end{pmatrix}$, $\vec e_\theta = \begin{pmatrix}\cos\theta\cos\varphi\\ \cos\theta\sin\varphi\\ -\sin\theta\end{pmatrix}$, $\vec e_\varphi = \begin{pmatrix}-\sin\varphi\\ \cos\varphi\\ 0\end{pmatrix}$

$\partial_r\vec e_r = 0$, $\partial_\theta\vec e_r = \vec e_\theta$, $\partial_\varphi\vec e_r = \sin\theta\,\vec e_\varphi$

Ellipsoid: $x = au\sin\theta\cos\varphi$, $y = bu\sin\theta\sin\varphi$, $z = cu\cos\theta$, $|J| = abc\,u^2\sin\theta$

## Ladungsverteilung

Ansatz: $\rho(\vec x) = \frac{1}{N} Q\,\delta(\ldots)\Theta(\ldots)$  $\int dV\rho(\vec x) \stackrel{!}{=} Q \Rightarrow N = \ldots$

Punktlad, Kugel $\rho(\vec x) = \frac{1}{r^2\sin\theta_0}Q\,\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(\theta-\theta_0)$

Kugeloberfläche $\rho(\vec x) = \frac{1}{4\pi R^2}Q\,\delta(R-r)$

Scheibe Zylinder $\rho(\vec x) = \frac{1}{\pi R^2}Q\,\delta(z)\Theta(R-r)$  $\vec j = \vec e_z\Theta(R-r)\frac{I}{\pi R^2}$

Zylinder 2a $\rho(\vec x) = \frac{1}{2\pi Ra}Q\,\delta(R-r)\Theta(z-a)\Theta(z+a)$

Kugel $\rho(\vec x) = \Theta(R-r)$  Ellipsoid: $\int_0^{2\pi}d\varphi\int_0^\pi\sin\theta\, d\theta\int_0^1 dr\, r^2\rho_0(1-r^2)abc$

Würfel $\rho(\vec x) = \rho_0\Theta(\vec x - \frac{a}{2})\Theta(\vec x + \frac{a}{2})$

Ellipsoid $\rho(\vec x) = \rho_0\Theta(1 - (x_1/a)^2 - (x_2/b)^2 - (x_3/c)^2)$

## Multipolentwicklung

$Q_{ij} = Q_{ji}$, $Q_{11}+Q_{22}+Q_{33} = 0$ 5 Freiheitsgrade

1) Ladungsverteilung bestimmen (ggf. $V_\rho = Q$)
2) Monopol $Q = \int dV' \rho(\vec r')$
3) Dipol $p_i = \int dV' \rho(\vec x')x'^i \Rightarrow$ in kgl. Koord. berechnen
4) Quadrupol $Q_{ij} = \int dV'(3x_i'x_j' - r'^2\delta_{ij})\rho(\vec r')$ $\Rightarrow$ 3 Parameter

Dipol, sphärisch: $q_{10} = q_{00}$ und $q_{11} = q_{1,-1}$

$\Rightarrow Q_{11} = \int dV'(3x'^2 - x'^2 - y'^2 - z'^2)\rho(\vec x') = 2I_1 - I_2 - I_3$

$$\begin{pmatrix} 2x'^2 - y'^2 - z'^2 & 3x'y' & 3x'z' \\ 3y'x' & 2y'^2 - x'^2 - z'^2 & 3y'z' \\ 3z'x' & 3z'y' & 2z'^2 - x'^2 - y'^2 \end{pmatrix}$$

$Q_{ij} = \sum_i q_i(3x_ix_j - r_i^2\delta_{ij})$, $\vec p = \sum_i q_i\vec r_i$

Symmetrieargumente: $\vec p = 0$, da symmetrisch $\rho(-x) = \rho(x)$

$\forall Q_{ij}$ mit $i\ne j \Rightarrow Q_{ij} = 0$ da symm.

$G(\vec r - \vec r') = \frac{1}{|\vec r - \vec r'|} = \sum_{l=0}^\infty\sum_{m=-l}^l \frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y_l^m(\theta,\varphi) Y_l^{m*}(\theta',\varphi')$

$\phi(\vec r) = \sum_{l=0}^\infty\sum_{m=-l}^{l}\frac{4\pi}{2l+1}\frac{1}{r^{l+1}}Y_l^m(\theta,\varphi)q_{lm}$, $q_{lm} = \int d^3r'\, r'^l Y_l^{m*}(\theta',\varphi')\rho(\vec r')$

Potential: Führender Multipol kann nicht durch Trafo verschwinden

$\phi(\vec x) = \phi_m(\vec x) + \phi_D(\vec x) + \phi_Q(\vec x)$  $\forall Q_{ij}$ mit $i\ne j \Rightarrow Q_{ij} = 0$ da symm

$\phi_m(\vec x) = \frac{Q}{r}$, $\phi_D(\vec x) = \frac{\vec p\vec x}{r^3}$, $\phi_Q(\vec x) = \frac12\sum_{ij}^3\frac{x_ix_j}{r^5}Q_{ij}$

## Energietransport & Poynting-Vektor

$w_{el} = \frac{E_iD^i}{8\pi}$, $w_{mag} = \frac{H_iB^i}{8\pi}$

M3 $\cdot H_i$, M4 $\cdot E_i \Rightarrow E_i\partial_{ct}D^i = \frac12\partial_{ct}(E_iD^i)$, M4 - M3

Poynting Vektor $P^i = \frac{c}{4\pi}\varepsilon^{ijk}E_jH_k$  $\partial_iP^i = -E_ij^i - \partial_t(w_{el} + w_{mag})$

$\vec S = \frac{c}{4\pi}\vec E\times\vec H$

Gauß $\frac{d}{dt}\int_V dV(w_{el} + w_{mag}) = -\int_V dV E_ij^i - \int_{\partial V} dS_iP^i$  Satz von Poynting

## Impulstransport & Poynting-Linearform

$U^2$ oder $RI^2$

$\frac{dp_i}{dt} \Rightarrow$ Lorentz, M1, M4 einsetzen, $+H_i\partial_jB^j$

$u = \frac{1}{8\pi}((\mathrm{Re}E)^2 + (\mathrm{Re}H)^2)$, $\vec S = \frac{c}{4\pi}\vec E\times\vec H$, $\vec g = \frac{1}{4\pi c}\vec D\times\vec B$

Poynting-Linearform $Y_i = \frac{1}{4\pi c}\varepsilon_{ijk}D^jB^k$

$\frac{d}{dt}\left(p_i + \int_V dV\, Y_i\right) = \int_V dV\,\partial_jT_i^j$

$T_i^j = \frac{1}{4\pi}\left(E_iD^j + H_iB^j - \frac12\delta_i^j(E_kD^k + H_kB^k)\right)$ Maxwell-Tensor

## Zeitabhängige Green-Fkt.

Lorenz-Eich: $\Box\phi = 4\pi\rho$, $\Box A_i = \frac{4\pi}{c}\gamma_{ij}j^j$

Helmholtz DGL: $\Box\psi(\vec r,t) = (\partial_{ct}^2 - \Delta)\psi(\vec r,t) = 4\pi q(\vec r,t)$

$\Rightarrow \Box G(\vec r - \vec r', t - t') = 4\pi\delta(t-t')\delta(\vec r - \vec r')$

$\Rightarrow \Box\psi(\vec r,t) = \int dV'\int dt'\,\Box G(\vec r - \vec r', t-t')q(\vec r',t') = 4\pi q(\vec r,t)$  mit + wäre akausal

$\Rightarrow G(\omega,\vec k) = 4\pi\frac{c^2}{\omega^2 - (ck)^2} \Rightarrow G_-(\vec r - \vec r', t-t') = \frac{1}{|\vec r - \vec r'|}\delta\left(t - t' - \frac{|\vec r - \vec r'|}{c}\right)$

$\psi(\vec r,t) = \int dV'\frac{1}{|\vec r - \vec r'|}\rho\left(\vec r', t - \frac{|\vec r - \vec r'|}{c}\right)$ retardiert  $t_{ret} = t - \frac1c|\vec x - \vec x'|$

## Helmholtz-Zerlegung

$\vec E_0(\vec r) = -\frac{1}{4\pi}\nabla\int d^3r'\frac{(\nabla\vec E_0)(\vec r')}{|\vec r - \vec r'|} + \frac{1}{4\pi}\nabla\times\int d^3r'\frac{(\nabla\times\vec E_0)(\vec r')}{|\vec r - \vec r'|}$

Longitudinal/transversal: $\vec E(\vec x) = \vec E_L(\vec x) + \vec E_T(\vec x)$

$\vec E_L(\vec k) = \frac{\vec k(\vec k\cdot\vec E)}{|\vec k|^2}$, $\vec E_T(\vec k) = -\frac{\vec k\times(\vec k\times\vec E)}{|\vec k|^2}$

E-Feld, Lichtwelle: $E_L = 0$, $E = E_T$

stat. EM-Felder: $E_T = 0$, $E = E_L$ (da $\nabla\times\vec E = -\dot{\vec B} = 0$)

B-Felder: $B = B_T$

$\rho_0 = \nabla\vec E_0$, $\vec B_0 = -\frac{c}{\omega}\nabla\times\vec E_0$  $\vec x = -\nabla\psi + \nabla\times\vec Q$  $\psi' = \psi + C$, $\vec Q' = \vec Q + \nabla\lambda$

## Spezielle Relativität

es gibt kein ausgezeichnetes Inertialsystem

$x^\mu = \binom{ct}{x}$, $u^\mu = \frac{dx^\mu}{d\tau}$  Galilei-Trafo: $x' = x - vt$, $t' = t$  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$

$ct' = \gamma(ct - \beta x)$, $x' = \gamma(x - \beta ct)$; $ct = \gamma(ct' + \beta x')$, $x = \gamma(x' - \beta ct')$  $\to \gamma \approx 1 + \frac{\beta^2}{2} + \ldots$

oder in Matrix-Schreibweise: $x'^\mu = \Lambda^\mu{}_\nu x^\nu$

$\binom{ct'}{x'} = \begin{pmatrix}\gamma & -\beta\gamma\\ -\beta\gamma & \gamma\end{pmatrix}\binom{ct}{x}$

Lorentz-Invarianten: Lorentz-Norm $s^2 = \eta_{\mu\nu}x^\mu x^\nu$, $\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$

$\tanh\psi = \beta$, $\gamma = \cosh\psi$, $\beta\gamma = \sinh\psi$

$v \ll c: \gamma\to1, \beta\to0$

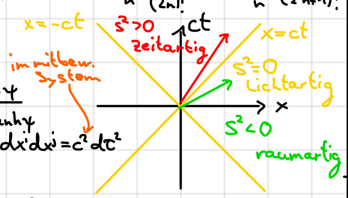
## Pauli-Matrizen

$\sigma^{(0)} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$, $\sigma^{(1)} = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $\sigma^{(2)} = \begin{pmatrix}0&1\\-1&0\end{pmatrix}$, $\sigma^{(3)} = \begin{pmatrix}0&1\\1&0\end{pmatrix}$

$\Lambda(\psi) = \exp(\psi\sigma^{(3)}) = \sum\frac{\psi^n}{n!}(\sigma^{(3)})^n = \sigma^{(0)}\sum\frac{\psi^{2n}}{(2n)!} + \sigma^{(3)}\sum\frac{\psi^{2n+1}}{(2n+1)!} = \begin{pmatrix}\cosh\psi&\sinh\psi\\\sinh\psi&\cosh\psi\end{pmatrix}$

$\Lambda(\psi)\Lambda(\varphi) = \Lambda(\psi+\varphi)$

$\tanh(\psi+\varphi) = \frac{\tanh\psi + \tanh\varphi}{1 + \tanh\psi\tanh\varphi}$




$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = c^2dt^2 - \gamma_{ij}dx^idx^j = c^2d\tau^2$

$d\tau = dt\sqrt{1 - \frac{1}{c^2}\left(\frac{d\vec x}{dt}\right)^2} = \frac{dt}{\gamma}$

Euler-Lagrange $\frac{d}{dt}\frac{\partial L}{\partial\dot x^\mu} - \frac{\partial L}{\partial x^\mu} = 0$

Bewegte Uhren gehen langsamer! Bewegte Maßstäbe scheinen verkürzt!  $L = -\frac{mc}{\gamma}$

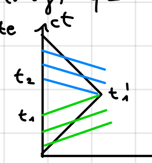
$s = \int ds = c\int^B d\tau \Leftrightarrow c\int^B dt\frac{1}{\gamma}$  $E_{rel} = mc^2\gamma$

relativistische EOM $\frac{d}{dt}(m\gamma\vec v) = q\vec E$

invariante $L \to$ kovariante

$E_x' = \gamma(E_x - \beta B_y)$, $E_y' = \gamma(E_y + \beta B_x)$, $E_z' = E_z$, $B_x' = \gamma(B_x + \beta E_y)$, $B_y' = \gamma(B_y - \beta E_x)$

## Zwillingsparadoxon



$\sigma^{(0)} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$, $\sigma^{(3)} = \begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\sigma^{(1)} = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $\sigma^{(2)} = \begin{pmatrix}0&1\\-1&0\end{pmatrix}$

$\gamma^2\beta^2 = \gamma^2 - 1$

## Wellengl. herleiten

1) $D^i = \varepsilon^{ij}E_j$ + isotrop / B analog
2) M3 bzw. M4 $|\partial_{ct}$
3) $\varepsilon^{ijk}\varepsilon_{klm}$ einsetzen
4) $\Box = \varepsilon\mu\partial_{ct}^2 - \Delta$

$\Lambda^\mu{}_\alpha\Lambda^\alpha{}_\beta = \delta^\mu_\beta$; $x^\mu \to \Lambda^\mu{}_\alpha x^\alpha$, $x_\mu \to \Lambda_\mu{}^\alpha x_\alpha$; $\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}$

$\nabla\vec v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$  $\mathrm{rot}(f\vec F) = f\,\mathrm{rot}\,\vec F + (\mathrm{grad}\,f)\times\vec F$

$\nabla\times\vec F = \begin{pmatrix}\frac{\partial F_3}{\partial x_2} - \frac{\partial F_2}{\partial x_3}\\ \frac{\partial F_1}{\partial x_3} - \frac{\partial F_3}{\partial x_1}\\ \frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2}\end{pmatrix}$  $\varepsilon^{\mu\nu\rho\sigma} = \begin{cases}1 & \text{für } (0,1,2,3) \text{ Perm}\\ -1 & \text{für ung. Perm}\\ 0 & \text{sonst}\end{cases}$

$F_{0i} = E_i$, $F_{ij} = -\varepsilon_{ijk}B^k$  $\partial_\mu\partial_\nu F^{\mu\nu} = 0$ antisym.

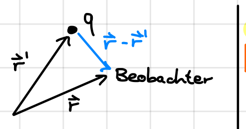
$\tilde G_{\mu\nu} = \begin{pmatrix}0 & -H^x & -H^y & -H^z\\ H^x & 0 & -D^z & D^y\\ H^y & D^z & 0 & -D^x\\ H^z & -D^y & D^x & 0\end{pmatrix}$  $G^{\mu\nu} = \begin{pmatrix}0 & D^x & D^y & D^z\\ -D^x & 0 & -H^z & H^y\\ -D^y & H^z & 0 & -H^x\\ -D^z & -H^y & H^x & 0\end{pmatrix}$

$\partial_\mu F^{\mu 0} \Rightarrow$ M1, $\partial_\mu \tilde F^{\mu\nu} \Rightarrow$ M3

$F_{\mu\nu} = \begin{pmatrix}0 & E_x & E_y & E_z\\ -E_x & 0 & -B_z & B_y\\ -E_y & B_z & 0 & -B_x\\ -E_z & -B_y & B_x & 0\end{pmatrix}$  $\tilde F^{\mu\nu} = \begin{pmatrix}0 & -B_x & -B_y & -B_z\\ B_x & 0 & E_z & -E_y\\ B_y & -E_z & 0 & E_x\\ B_z & E_y & -E_x & 0\end{pmatrix}$

## Integrale

$\cos(n\pi) = (-1)^n$, $\sin(n\pi) = 0$




$\int_0^\pi\sin^3\theta\,d\theta = \frac43$  $\int dx\frac{1}{\sqrt{1-x^2}} = \arcsin(x)$

$\int_0^\pi\sin\theta\cos^2\theta\,d\theta = \frac23$  $\int dx\frac{1}{\sqrt{1+x^2}} = \mathrm{arcsinh}(x)$

$\int_0^{2\pi}\cos^2\varphi\,d\varphi = \int_0^{2\pi}\sin^2\varphi\,d\varphi = \pi$  $\cosh^2(x) - \sinh^2(x) = 1$

$\int\cos^2(kx)dx = \frac{x}{2} + \frac{\sin(2kx)}{4k} + C$  $\langle\cos^2\rangle = \frac12$

$\int\sin^2(kx)dx = \frac{x}{2} - \frac{\sin(2kx)}{4k} + C$  $\int_{-\infty}^\infty dx\,e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$

## Delta-Fkt.

$\int f(x)\delta(x)dx = f(0) = \int f(x)\delta(-x)dx$

$\int f(x)\delta(x - x_0) = f(x_0)$  $\mathcal F(e^{ikx}) = 2\pi\delta_D(k - k_0)$

$\int f(x)\delta(|k|x) = \frac{1}{|k|}f(0)$

Taylor: $G(\vec r - \vec r') = G|_{\vec r=0} + \partial_iG|_{\vec r=0}x^i + \frac{1}{2!}\partial_i\partial_jG|_{\vec r=0}x^ix^j$

$\Rightarrow\phi(\vec r) = \int_{\mathbb R^3}d^3r'\sum_{n=0}^\infty\frac{\partial_{i_1}\ldots\partial_{i_n}}{n!}G|_{\vec r=0}(r')^{i_1}\ldots(r')^{i_n}\rho(\vec r')$

Kurvenintegrale $\int_C dz\,f(z) = \int dt\,\dot\gamma(t)f(\gamma(t))$, Bsp. $\gamma(t) = e^{2\pi it}$, $t\in[0,1]$

Residuen n-ter Ordnung: $\mathrm{Res}(f,z_0) = \frac{1}{(n-1)!}\lim_{z\to z_0}\frac{d^{n-1}}{dz^{n-1}}[(z-z_0)^nf(z)]$

$\binom{i}{j} = \frac{i!}{j!(i-j)!}$  $\Delta\phi = \frac{1}{r}\partial_r(r\partial_r\phi) + \frac{1}{r^2}\partial_\varphi^2\phi$

$\mathrm{Re}\,\vec a = \frac{\vec a + \vec a^*}{2}$  $(\mathrm{Re}\,\vec a)^2 \ne \mathrm{Re}\,\vec a^2$

Ansätze: $\ddot x - w^2x = 0 \Rightarrow x(t) = C_1e^{wt} + C_2e^{-wt}$; $\ddot x + w^2x = 0 \Rightarrow x(t) = C_1e^{iwt} + C_2e^{-iwt}$

inhom. Wellengl. in Materie:

M3 $|\partial_{ct}$, M4 für $\partial_{ct}E$ ein $\Rightarrow \Box B^i = \frac{4\pi}{c}\mu\varepsilon^{ijk}\partial_jj_k$

M4 $|\partial_{ct}$, M3 für $\partial_{ct}B$ ein $\Rightarrow \Box E^i = -\frac{4\pi}{c\varepsilon}\partial^i\rho - \frac{4\pi\mu}{c}\partial_{ct}j^i$

M1 $D^i = \varepsilon E^i$, $E^i = -\partial^i\phi - \partial_{ct}A^i$, Lorentz: $\partial_iA^i = -\mu\varepsilon\partial_{ct}\phi \Rightarrow \Box\phi = 0$

M4 $H_k = \frac{1}{\mu}B_k$, $E^i = -\partial^i\phi - \partial_{ct}A^i$, $B_k = \varepsilon_{klm}\partial^lA^m$, Lorentz $\Rightarrow \Box A^i = 0$

$\exp(x) = \sum_{n=0}^\infty\frac{x^n}{n!}$, $\cos(x) = \sum_{n=0}^\infty\frac{x^{2n}}{(2n)!}(-1)^n$, $\sin(x) = \sum_{n=0}^\infty\frac{x^{2n+1}}{(2n+1)!}(-1)^n$, $\sinh(x) = \frac12(e^x - e^{-x})$

$\cosh(x) = \frac12(e^x + e^{-x})$, $\sinh(x) = \sum_{n=0}^\infty\frac{x^{2n+1}}{(2n+1)!}$, $\cosh(x) = \sum_{n=0}^\infty\frac{x^{2n}}{(2n)!}$

Zeitdilatation $\Delta x' = 0$ / $\Delta x = vt \Rightarrow T = T_0\gamma$  $a_\mu a^\mu > 0$: zeitartig

Längenkontraktion $\Delta t = 0 \Rightarrow L = \frac{L_0}{\gamma}$  $a_\mu a^\mu < 0$: raumartig

relativistische Energie $E = mc^2\gamma$  $a_\mu a^\mu = 0$: lichtartig

$\gamma > 1$

$\vec B = \int dV'\frac{\vec j(\vec r')\times(\vec r - \vec r')}{|\vec r - \vec r'|^3}$  M1 $\oint\vec E\,d\vec A = 4\pi\int dV\rho(\vec x)$

M2 $\oint\vec B\,d\vec A = 0$

$\vec E = \int dV'\frac{\rho(\vec r')(\vec r - \vec r')}{|\vec r - \vec r'|^3}$  M3 $\oint_{\partial A}\vec E\,d\vec s = -\int_A\frac{\partial\vec B}{\partial t}d\vec A$

M4 $\oint_{\partial A}\vec H\,d\vec s = \int_A\vec j\,d\vec A + \int_A\frac{\partial\vec D}{\partial t}d\vec A$

## Fouriertrafo

$f(x) = \begin{cases} y & y = x + 3k \\ y-1 & y = x + 3k \end{cases}$

$f(x) = \frac{a_0}{2} + \sum_{k=1}^\infty\left(a_k\cos\left(\frac{2\pi}{T}kx\right) + b_k\sin\left(\frac{2\pi}{T}kx\right)\right)$  $b_k = \frac{2}{T}\int_{-T/2}^{T/2}f(x)\sin\left(\frac{2\pi}{T}kx\right)dx$

$a_0 = \frac{2}{T}\int_{-T/2}^{T/2}f(x)dx$, $a_k = \frac{2}{T}\int_{-T/2}^{T/2}f(x)\cos\left(\frac{2\pi}{T}kx\right)dx$

## Kovariante Elektrodynamik

Bew.gl. $\frac{d}{dt}(\gamma m\vec v) = \vec F = q\vec E + \frac{q}{c}\vec v\times\vec B$  Impuls $p^\mu = \binom{m\gamma c}{\vec p}$

Vierergradient $\partial_\mu = \frac{\partial}{\partial x^\mu} = (\partial_{ct}, \nabla)$ $\Rightarrow \Box = \partial_{ct}^2 - \nabla^2 = \partial^\mu\partial_\mu$

Vierergeschw. $u^\mu = \frac{dx^\mu}{d\tau} = \binom{\gamma c}{\gamma\vec v}$  $u^\mu u_\mu = c^2$

Viererwellenvekt. $k^\mu = \binom{\omega/c}{\vec k} \Rightarrow k_\mu k^\mu = \left(\frac{\omega}{c}\right)^2 - k^2$

Viererstromdichte $j^\mu = \binom{c\rho}{\vec j} \Rightarrow \partial_\mu j^\mu = \partial_t\rho + \nabla\vec j = 0$

Viererpotential $A_\mu = (\phi, -\vec A)$, $A^\mu = \binom{\phi}{\vec A} \Rightarrow$ Eichfreiheit: $A_\mu \to A_\mu + \partial_\mu\chi$

Feldstärketensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\Rightarrow$ Lorentz $\eta^{\mu\nu}\partial_\mu A_\nu = \partial_\mu A^\mu = 0$

$F^{\mu\nu} = \eta^{\mu\alpha}\eta^{\nu\beta}F_{\alpha\beta}$  $\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$

Maxwell-Gleichungen: homogen $\partial_\mu\tilde F^{\mu\nu} = 0$; inhomogen $\partial_\mu F^{\mu\nu} = \frac{4\pi}{c}j^\nu$; in Materie $\partial_\mu G^{\mu\nu} = \frac{4\pi}{c}j^\nu$

wave eq. $\Box F_{\alpha\beta} = 0 \Rightarrow F_{\alpha\beta} \propto \exp(\pm ik_\mu x^\mu)$, $\eta^{\mu\nu}k_\mu k_\nu = 0 \Rightarrow \omega = \pm ck$

nicht-Vakuum: $\Box F_{\alpha\beta} = \frac{4\pi}{c}\eta^{\mu\nu}(\partial_\alpha\eta_{\beta\mu}j_\nu - \partial_\beta\eta_{\alpha\mu}j_\nu)$

in Lorenzeichung: $\partial_\mu F^{\mu\nu} = \frac{4\pi}{c}j^\nu \Rightarrow \Box A^\nu = \frac{4\pi}{c}j^\nu$

## Energie-Impuls-Tensor

$T^\mu{}_\nu = \frac{1}{4\pi}\left(F^{\mu\alpha}F_{\alpha\nu} + \frac14\delta^\mu_\nu F^{\alpha\beta}F_{\alpha\beta}\right)$

$T_\nu{}^\mu = \begin{pmatrix} w_{el} + w_{mag} & Y_i \\ P^i & T_i^j\end{pmatrix}$

Wenn in I: $\vec B = 0$ $\Rightarrow \vec E\vec B = \text{inv.} \Rightarrow \vec E \perp \vec B$ in I'; $\vec E^2 - \vec B^2 = \vec E'^2 - \vec B'^2 \ge 0$; $\vec E = 0$ und $\vec B \ne 0$

$\vec E\vec B = \vec E'\vec B'$; $|\vec E||\vec B|\cos\theta = |\vec E'||\vec B'|\cos\theta'$ gleiches Vorzeichen

$\partial_\mu\left(\frac{\partial\mathcal L}{\partial(\partial_\mu A_\nu)}\right) - \frac{\partial\mathcal L}{\partial A_\nu} = 0$ Lagrange 2. Art

$H = \dot x\frac{\partial L}{\partial\dot x} - L$

## Lagrange

L invariant & quadratisch $\Rightarrow$ EOM kovariant & linear

E-Statik: $\mathcal L(\phi, \partial_i\phi) = \frac14\gamma^{ij}\partial_i\phi\partial_j\phi + 4\pi\rho\phi$  $S = \int d^3x\,\mathcal L$

E-Dynamik: $\mathcal L(A_\mu, \partial_\mu A_\nu) = \frac14F^{\mu\nu}F_{\mu\nu} + \frac{4\pi}{c}A_\mu j^\mu$  $S = \int d^4x\,\mathcal L$  $ds^2 = (c\,d\tau)^2$

## Lorentz-Invarianten

rel. Bew. $L(\dot x) = -\frac{mc}{\gamma}$  $S = \int_A^B ds = \int_A^B\sqrt{\eta_{\mu\nu}dx^\mu dx^\nu} = \int_A^B\sqrt{\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\,d\tau$

$F^{\alpha\beta}G_{\alpha\beta} = E_iD^i - H_iB^i = F_{\mu\nu}G^{\mu\nu}$

$\tilde F^{\alpha\beta}F_{\alpha\beta} \propto E_iB^i$  gleich im Vakuum

$\tilde G_{\mu\nu}G^{\mu\nu} \propto H_iD^i$

$\tilde F^{\alpha\beta} = \frac12\varepsilon^{\alpha\beta\mu\nu}F_{\mu\nu} \leftrightarrow F_{\mu\nu} = -\frac12\varepsilon_{\mu\nu\alpha\beta}\tilde F^{\alpha\beta}$

$\tilde G_{\mu\nu} = \frac12\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta} \leftrightarrow G^{\alpha\beta} = -\frac12\varepsilon^{\alpha\beta\mu\nu}\tilde G_{\mu\nu}$

## Rotierende Kugel

$\vec A = \frac{1}{4\pi}\int dV'\frac{\vec j(\vec r')}{|\vec r - \vec r'|}$

$\vec v = \vec\omega\times\vec r$  $\vec j = \delta(r - R)\frac{Q}{4\pi R^2}\vec\omega\times\vec r$

$\vec r$ entlang z-Achse wählen mit $\theta'$ zwischen $\vec r$ und $\vec r'$ $\hookrightarrow \vec r\vec r' = rr'\cos\theta'$

$\frac{1}{|\vec r - \vec r'|} = \frac{1}{\sqrt{r^2 - 2rr'\mu + r'^2}}$ mit $\mu = \cos\theta'$

Fallunterscheidung bei $\sqrt{(r-R)^2} \Rightarrow (r-R)$ für $r > R$, $(R-r)$ für $r < R$

$\vec a\times\vec b = |a||b|\sin\theta$

$\vec B_{Dipol} = \frac{1}{r^5}(3\vec r(\vec\mu\cdot\vec r) - r^2\vec\mu)$

## Ableitungen:

$\nabla\times\frac{\vec x}{|\vec x|^3} = 0$  $\nabla\cdot\frac{\vec x}{|\vec x|^3} = 4\pi\delta(\vec x)$  $\partial_i\left(\frac{x_i - x_i'}{|\vec x - \vec x'|^3}\right) = -4\pi\delta(\vec x - \vec x')$

$\nabla\frac{1}{|\vec x|} = -\frac{\vec x}{|\vec x|^3}$  $\partial_j\frac{1}{|\vec x - \vec x'|} = -\frac{(x_j - x_j')}{|\vec x - \vec x'|^3}$

$\nabla\frac{\vec r}{r^3} = \left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right)\frac{1}{r^3} + \vec r\frac{\partial}{\partial x_i}\left(\frac{1}{r^3}\right) = \frac{3}{r^3} - \frac{3}{r^3} = 0$

$(\vec\omega\nabla)\frac{\vec r}{r^3} = \frac{1}{r^3}(\vec\omega\nabla)\vec r + \vec r(\vec\omega\nabla)\frac{1}{r^3} = \frac{\vec\omega}{r^3} - \frac{3\vec r(\vec\omega\vec r)}{r^5}$

$\frac{\partial}{\partial x_k}\left(\frac{1}{r^s}\right) = \frac{\partial}{\partial r}(r^{-s})\frac{\partial r}{\partial x_k} = -sr^{-s-1}\frac{x_k}{r} = -s\frac{x_k}{r^{s+2}}$

Telegraph.gl. E: M3 $|\mu_{il}\varepsilon^{mnl}\partial_n$, M4 ein, $\partial^kE_k = 0$, isotrop. Med.

B: M3 $\partial_{ct}$, M4 $|\mu_{il}\varepsilon^{mnl}\partial_n$, M3 ein, $\partial^kB_k = 0$  $j^i = \sigma^{ij}E_j = \sigma E^i$

Rel. Beweg. $\frac{d}{dt}(m\gamma\vec v) = q\vec E + \frac{q}{c}\vec v\times\vec B$

$\cosh(\psi) = 1 + \frac{\psi^2}{2}$

1) $\vec F = \frac{q}{c}\vec v\times\vec B$

## Lagrange herleiten

$\delta S = \delta\int d^3x\,\mathcal L = \int d^3x\left(\frac{\partial\mathcal L}{\partial\phi}\delta\phi + \frac{\partial\mathcal L}{\partial(\partial_i\phi)}\delta\partial_i\phi\right) = 0$

1) $\delta\partial_i\phi = \partial_i\delta\phi$
2) part. Integration

I) $\varepsilon^{ijk}\partial_jV_k = 0 \Rightarrow \exists\Phi: V_k = -\partial_k\Phi$

II) $\partial_iV^i = 0 \Rightarrow \exists A_k: V^i = \varepsilon^{ijk}\partial_jA_k$

Wellen $A = |A|e^{i\phi_A}$, $B = |B|e^{i\phi_B}$  $\frac{A}{B} = \pm\frac{|A|}{|B|}$

Linear: 1) $\phi_A = \phi_B + n\pi$, $n\in\mathbb N$

Zirkular: 1) $\phi_A - \phi_B = \pm\frac{\pi}{2}$, $n\in\mathbb N$; 2) $|A| = |B|$

elliptisch: 1) $\phi_A - \phi_B \ne \frac{\pi}{2}$, $n\in\mathbb N$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x}$, $\frac{\partial^2f}{\partial x^2} = \frac{\partial^2f}{\partial u^2}\frac{\partial u}{\partial x}\frac{\partial u}{\partial x}$

Lichtkegelkoordinaten: $a = x + ct$, $b = x - ct$, $\Box\psi = 4\frac{\partial^2}{\partial a\partial b}\psi$

Separationsansatz $\frac{\ell(t)}{\ell(t)} = 1$, $\frac{\ell(x)}{\ell(x)} = \alpha$, Konstanten in räumliche DGL

div(M4)

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$  $e^{ix} = \cos x + i\sin x$

$\sin(x-y) = \sin(x)\cos(y) - \cos(y)\sin(x)$  $e^{-ix} = \cos x - i\sin x$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$  $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$

$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$  $\cos x = \frac12(e^{ix} + e^{-ix})$

$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\sin^2(x) = \frac12(1 - \cos(2x))$, $\cos^2(x) = \frac12(\cos(2x) + 1)$

Legendre-Polynome: $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac12(3x^2 - 1)$, $P_3(x) = \frac12(5x^3 - 3x)$

DGL-Checkliste
1) gewöhnlich vs. partiell
2) homogen vs. inhomogen
3) linear vs. nichtlinear
4) Ableitungsordnung

$\exp(A)\exp(B) = \exp(A+B)\exp\left(-\frac12[A,B]\right)$

## Vektoranalysis

$\mathrm{div}(\mathrm{rot}) = 0$

$\mathrm{rot}(\mathrm{rot}\,\vec x) = \mathrm{grad}(\mathrm{div}\,\vec x) - \Delta\vec x$  $\vec m(\vec r) = \frac{1}{2}\int\vec r'\times\vec j(\vec r')\,d^3r'$

$\mathrm{div}(\phi\vec x) = \phi\,\mathrm{div}\,\vec x + \vec x\cdot\mathrm{grad}\,\phi$

$\mathrm{rot}(\phi\vec x) = \phi\,\mathrm{rot}\,\vec x + (\mathrm{grad}\,\phi)\times\vec x$

$\mathrm{div}(\vec x\times\vec y) = \vec y\,\mathrm{rot}\,\vec x - \vec x\,\mathrm{rot}\,\vec y$

$\mathrm{rot}(\vec x\times\vec y) = \vec x\,\mathrm{div}\,\vec y - \vec y\,\mathrm{div}\,\vec x + (\vec y\cdot\mathrm{grad})\vec x - (\vec x\cdot\mathrm{grad})\vec y$

$\mathrm{grad}(\vec x\cdot\vec y) = \vec x\times\mathrm{rot}\,\vec y + \vec y\times\mathrm{rot}\,\vec x + (\vec y\cdot\mathrm{grad})\vec x + (\vec x\cdot\mathrm{grad})\vec y$

## Legendre

$a_\ell = \frac{2\ell+1}{2}\int d\cos\theta\,P_\ell(\cos\theta)f$

E-Feld, $Q_{ij}$ Ursprung: $E = -\nabla\phi_Q = \frac12\left(\frac{\sum Q_{ij}x_j}{r^5}5\vec x - 2\frac{Q\vec x}{r^5}\right)$

## Abhängigkeiten

Dipolkraft: $\vec F = +\nabla(\vec p\vec E)$

$\vec F = \nabla(\vec B\vec m)$, $\phi = \frac{q}{|\vec r - \vec r'|}$, $\vec M = \vec m\times\vec B$

Monopol: $\phi\sim 1/r$, $E\sim 1/r^2$, $F\sim 1/r^2$

Dipol: $\phi\sim 1/r^2$, $E\sim 1/r^3$, $F\sim 1/r^4$

Quadrupol: $\phi\sim 1/r^3$, $E\sim 1/r^4$

## Vektorpotentiale konstruieren

1) $\mathrm{rot}\,\vec A' = \vec B \Rightarrow \varepsilon^{ijk}\partial_jA_k' = \varepsilon^{2jk}\partial_2A_k' + \varepsilon^{3jk}\partial_3A_k' \stackrel{!}{=} B^i$
2) $\partial_2A_1' \stackrel{!}{=} 0$
3) $\vec A' = \vec A + \nabla\chi \Rightarrow \vec A = \vec A' - \nabla\chi$

$\omega^2 = \frac{k^2c^2}{\mu\varepsilon} \Rightarrow v_{Ph} = \frac{c}{\sqrt{\mu\varepsilon}}$  $R = \frac{l}{\sigma A}$

## Tricks:

- Ableiten für harmonischen Osz.
- $\gamma$ = const für B-Feld
- Annahme: $E_i = |E|e^{i\delta}$
- Wellengl. herleiten $\Rightarrow$ M3, M4 $|\partial_{ct}$, isotrop, $H \sim B$
- Felder & Abl. im $\infty \to 0$
- Unabh. vom Ursprung: $\vec x' = \vec x + \vec a$ $\Rightarrow$ $|\vec x'|^2 = |\vec x|^2 + 2\vec x\vec a + |\vec a|^2$

Eichtrafo: $\vec A \to \vec A' = \vec A + \nabla\chi$, $\vec A \to \vec A' = \vec A - \nabla\chi$